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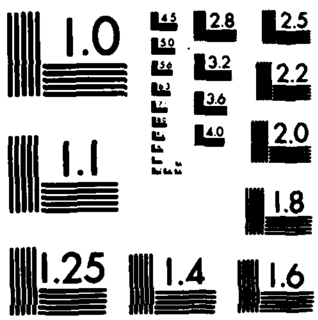

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Technical Report
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Design of Microwave Beam-Switching Networks

M.L. Burrows

5 December 1983

Lincoln Laboratory
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LEXINGTON, MASSACHUSETTS

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This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

Thomas J. Alpert

Thomas J. Alpert, Major, USAF
Chief, ESD Lincoln Laboratory Project Office

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

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M.L. BURROWS

Group 61

TECHNICAL REPORT 639

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LEXINGTON MASSACHUSETTS

Abstract

An investigation of RF beam-switching networks for creating, with a multiple-beam antenna (MBA), a set of electronically steerable antenna beams, shows that there are three main classes of network. Taking the case of a 61-beam MBA with 8 simultaneously steered beams as an example, we can describe these classes as a) networks which can connect any one of the 8 output ports to any one of the 61 beam feeds, b) networks which can connect the 8 output ports to any set of 8 beam feeds selected from the 61, but with a constraint imposed on the order in which the 8 ports are connected to the 8 selected feeds, and c) networks in which the set of 8 beam feeds cannot be selected arbitrarily - some fraction of the total number of conceivable interconnections cannot be completed.

Networks exist in each of the three classes having very similar traffic handling performance and yet requiring a total number of switches which is very different from one class to the next. Their number is 907, 387, and 175 for the unconstrained, order-constrained and selection-constrained networks, respectively, needed to do the 61-to-8 switching job described above.

The report examines the design, performance and complexity of such networks for general N and M. Included are two further measures of complexity, as well as the switching algorithm and the effect of non-uniform traffic. The results are presented as graphs, tables and formulas.

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I. INTRODUCTION

An investigation of RF beam-switching networks for creating, with a multiple-beam antenna (MBA), a set of electronically steerable antenna beams, shows that there are three main classes of network. Taking the case of a 61-beam MBA with 8 simultaneously steered beams as an example, we can describe these classes as a) networks which can connect any one of the 8 output ports to any one of the 61 beam feeds, b) networks which can connect the 8 output ports to any set of 8 beam feeds selected from the 61, but with a constraint imposed on the order in which the 8 ports are connected to the 8 selected feeds, and c) networks in which the set of 8 beam feeds cannot be selected arbitrarily - some fraction of the total number of conceivable interconnections cannot be completed.

Networks exist in each of the three classes having very similar traffic handling performance and yet requiring a total number of switches which is very different from one class to the next. Their number is 907, 387, and 175 for the unconstrained, order-constrained and selection-constrained networks, respectively, needed to do the 61-to-8 switching job described above.

In the following sections, the design of the general N-to-M switching network of each class is examined. The hardware parameters considered are (in addition to the total number of switches) the maximum number of switches in series in any signal path, and the number of possible signal paths physically in parallel. The network design procedure and switching algorithm are described, and the traffic handling capacity is evaluated. The superior performance of one type of selection-constrained network, (the "merged"), under non-uniform traffic conditions, compared with that of another type (the "discrete") is demonstrated.

It should be noted that the report is devoted solely to evaluating some particular parameters associated with a waveguide switch network. Not considered are the trade-offs between switching at RF rather than at IF or the incorporation of redundancy. The report addresses only the more narrow problem of designing an N-to-M waveguide switching network when N and M are given.

However, the possibility of the RF/IF trade-off is an important consideration in the design of the network. In particular, it allows a greatly simplified and less lossy RF switching network to be used, by relying on the IF (or lower-frequency) switching to reorder the set of beam feeds selected, but not placed in proper order, by the RF switching.

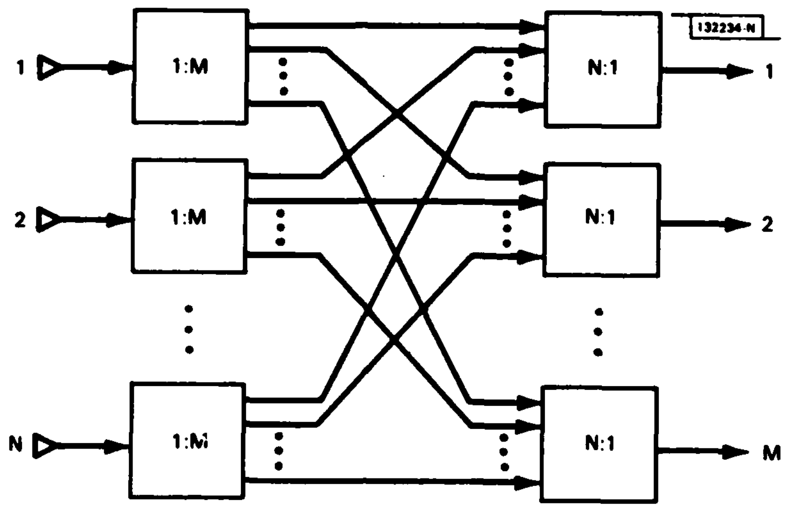

Fig. 1. The direct implementation of the general unconstrained switching network.

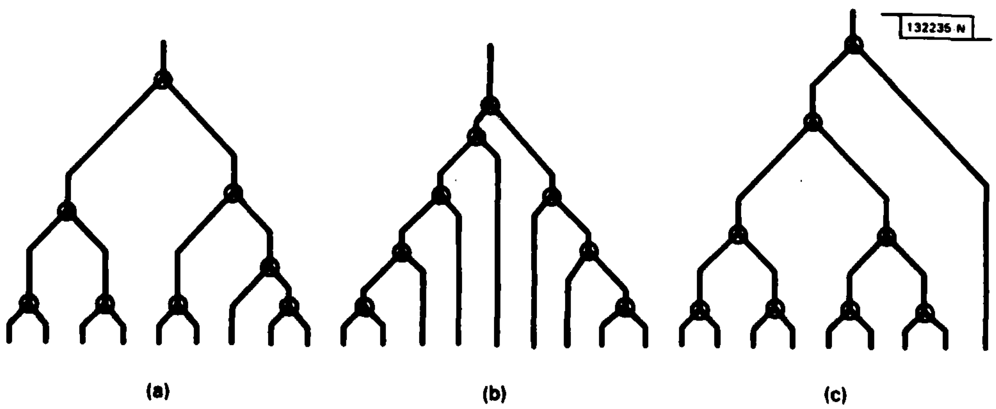

Fig. 2. Circuit diagrams of a 1-to-9 subunit showing the preferred design (a) and two more lossy designs (b) and (c).

II. UNCONSTRAINED NETWORKS

Conceptually, the simplest switching network capable of connecting M output ports to any M of N beam feeds, in any order, is that shown in Figure 1. It is assembled from a total of N + M separate sub units, each of which is a switching network capable of connecting a single port on one side to any one of the n ports (where n stands for N or M) on the other side. If we denote by S the number of switches needed by the subunit to perform its function, and by L the maximum number of switches in series in any signal path through the subunit, then for a subunit design which minimizes the numbers S and L, they are given by

$$S = n - 1 \qquad (1)$$

$$L = 1 + \mathrm{Int}\{\log_2\{(n - 0.1\}, \qquad (2)$$

where Int stands for "integer part of" and $\mathrm{Int}\{-|x|\} = -\mathrm{Int}\{|x|+1\}$.

The general configuration of the subunit waveguide circuit is shown in Figure 2(a). Figure 2(b) shows an alternative configuration which increases unnecessarily the maximum number of switches in any one signal path, and Figure 2(c) a configuration which increases unnecessarily the number of signal paths having the maximum number of switches in series. Both the alternative configurations impart more signal loss than is necessary in some of the signal paths.

The switches are denoted by small circles at each three-way waveguide junction. Physically each switch could be a ferrite two-state circulator. It can be used to route signals from a selected input port to the output port, or vice versa. However, it should be noted that the circulator is a non-reciprocal device. The states of all the switches in the signal path would have to be reversed if the signal direction is reversed. Figure 3 shows the signal flow direction for the two states of a switch.

From Figure 1 and equations (1) and (2) we find that for the direct implementation of the unconstrained N-to-M network, the maximum number L of switches in any one signal path and the total number S of switches required are given by

$$S = 2NM - M - N \qquad (3)$$

$$L = 2 + \mathrm{Int}\{\log_2(N-0.1)\} + \mathrm{Int}\{\log_2(M-0.1)\}. \qquad (4)$$

For convenience, tables of these formulas are presented in the Appendix.

For large M and N, (3) shows that the number of switches required for this unconstrained N-to-M switching network is itself very large. For example, if N=61 and M=16, then 1875 switches are required. The maximum number of switches in series in any signal path, for the same example, is not less than 10. If the insertion loss of a single switch is 0.25 dB, therefore, the total insertion loss cannot be less than 2.5 dB (for that signal path through the whole network having the maximum number of switches).

Also of great significance for this unconstrained network is the size and complexity of the waveguide plumbing job. Reference to Figure 1 shows that the number of possible signal paths in parallel between the front and back ranks of subunits is given by NM, of which only M are in use at any one

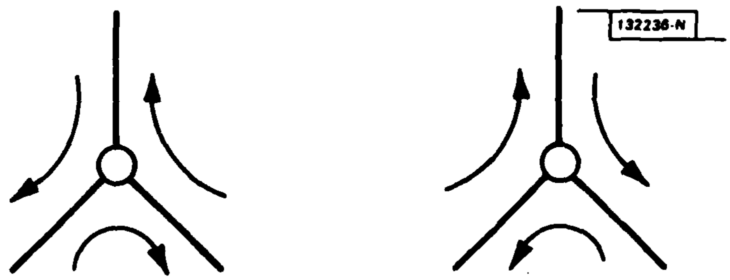

Fig. 3. The two states of the latching ferrite waveguide switch, showing the paths of easy signal flow.

time. For the example of the previous paragraph (N=61, M=16), the product NM is 976, and only 16 are in use at any one time. Thus the unconstrained network having the configuration shown in Figure 1 makes very inefficient use of its hardware.

The direct implementation shown in Figure 1 is not the only possible way of connecting the M output ports in any order to any M of N beam feeds, however. There exist conceptually more complicated networks which can do the job with a fewer total number of switches, but they also seem to place a considerably larger number of switches in series in any one signal path.

One such network is a compound arrangement of two networks in series, as shown in Figure 4. The first network is of the order-constrained type discussed in the next section. It enables any M of the N beam feeds to be connected to the M output ports. The second network is the directly implemented unconstrained type shown in Figure 1 and has M input ports and M output ports. It overcomes the ordering constraint imposed by the first network by allowing an arbitrary reordering to be carried out before the final M output ports. As the discussion in the next section makes clear, the number of switches required for the order-constrained N-to-M "selection" network is not so readily evaluated as for the unconstrained type. It is, however substantially less, and can more than make up for the additional switches required for the unconstrained "reordering" network.

For example, if N=61 and M=16 as before, then the order-constrained selection network requires 459 switches and has a maximum of 10 switches in series in any of its signal paths. The unconstrained reordering network

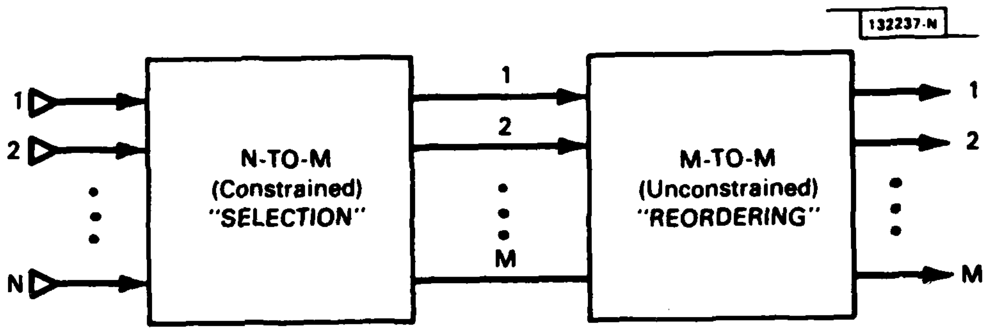

Fig. 4. Compound implementation of the general unconstrained N-to-M switching network.

following it requires, from (3), 480 switches and has, from (4), a maximum of 8 switches in series in any of its signal paths. Thus the complete compound network requires 939 switches and has a maximum of 18 switches in series in any of its signal paths. The compound network has therefore achieved a substantial reduction in the total number of switches required (939 instead of 1875), but at the price of a substantial increase in the maximum number of switches in any signal path (18 instead of 10).

The general formulas for S and L for the order-constrained network are presented in the next section. Adding the values of S and L obtained from those formulas to the values obtained from (3) and (4), with N=M, we can evaluate S and L for the complete compound network. The results are presented in tabular form in the Appendix. The formulas for the order-constrained network need a lengthy description and so their presentation is left for the next section.

Another feature of concern is the plumbing complexity. For the direct implementation of the unconstrained network, the total number P of possible signal paths physically existing in parallel is given by NM. For the compound implementation, P is N, if M=1, and $\mathrm{Max}\{2(N-1),M^2\}$ otherwise, and for the order-constrained network it is N, if M=1, and 2(N-1) otherwise. These expressions are presented in tabular form in the Appendix. Thus the plumbing complexity can be very much less for the compound implementation and for the order-constrained network. The sketches of the three circuits shown in Figure 5 for the simple case N=6, M=3 illustrate clearly the differences in complexity. The quantity P is the maximum number of signal paths than can be cut by a plane perpendicular to the direction of signal flow.

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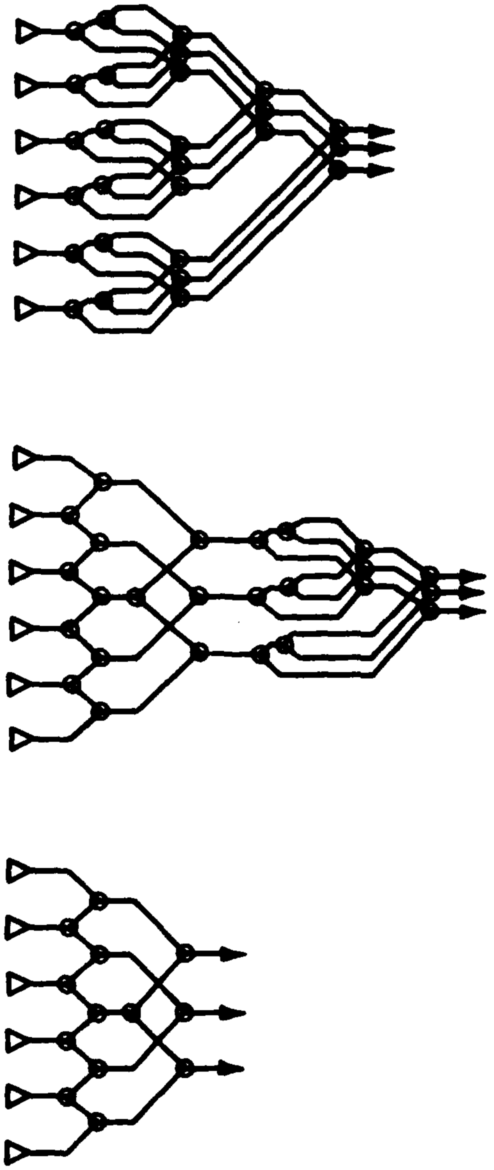

(a) DIRECT IMPLEMENTATION, UNCONSTRAINED NETWORK
S = 27, L = 5, P = 18

(b) COMPOUND IMPLEMENTATION, UNCONSTRAINED NETWORK
S = 25, L = 8, P = 10

(c) ORDER-CONSTRAINED NETWORK
S = 13, L = 4, P = 10

Fig. 5. Simple examples, for N=6 and M=3, of the two types of unconstrained network, and of the order-constrained type. P is the number of possible signal paths physically in parallel.

Table I presents a comparison of the total number S of switches, the maximum number L of switches in any one signal path and the total number P of possible signal paths physically in parallel, for the two implementations (the direct and the compound) of the unconstrained N-to-M switching network, for some selected values of N and M. Also included, for completeness, are the same data for the order-constrained network described in the next section.

The data in Table I show that if N is much larger than M and if M itself is much larger than 1, then the compound implementation of the unconstrained network, in comparison with the direct implementation, achieves a substantial reduction in the total number of switches required. However, if N is not much larger than M, the reduction is comparatively minor, and in all cases the maximum number of switches in series in any signal path is substantially larger in the compound implementation. The plumbing complexity, measured by P, is significantly less in the compound implementation, but it still remains high for the higher values of M. The contrast between both implementations of the unconstrained network, on the one hand, with the order-constrained network, on the other, is very marked. The values of S and P required by the order-constrained network are very much less than those required by the other two in all but the simplest examples (and they are still no greater in those cases), and L is always no greater, and in many cases substantially less.

These results lead to the conclusion that there is great advantage to be gained from a system design for which the order-constrained type of network is acceptable. The uncontrained networks quickly become intractable in number of switches and plumbing complexity as the number of input and output ports rises.

TABLE I

S, L, AND P FOR THREE NETWORKS

		Unconstrained						Order-Constrained		
		Direct Implementation			Compound Implementation					
N	M	S	L	P	S	L	P	S	L	P
10	2	28	5	20	28	7	18	24	5	18
10	5	85	7	50	73	12	25	33	6	18
10	8	142	7	80	142	10	64	30	4	18
25	5	220	8	125	158	14	48	118	8	48
25	12	563	9	300	403	16	144	139	8	48
25	18	857	10	450	733	16	324	121	6	48
61	8	907	9	488	499	15	120	387	9	120
61	30	3569	11	1830	2205	20	900	465	10	120
61	54	6473	12	3294	6061	18	2916	337	6	120

The algorithm for controlling the switches of the unconstrained networks in straightforward for the directly implemented type. Each subunit is either an N-to-1 or a 1-to-M subnetwork of the type sketched in Figure 2(a), and the switch settings within the subunits associated with beam feed n (or output port m) depend only on the designation of the output port (or beam feed) to which beam feed n (or output m) is to be connected. Thus to connect beam feed n to output port m, the switch settings in the subunit associated with beam feed n are determined solely by the number n. And if n is expressed in the binary form $n = b_q \ldots b_2 b_1$, where the b_r are the individual bits of the binary number n, then the settings of successive ranks of switches, starting from the single input port, are given directly by the b_r, starting from b_q, the most significant bit.

The same approach applies to the reordering section of the compound implementation of the unconstrained network, because it is an M-to-M directly-implemented unconstrained network. The selection section is an order-constrained network, the switching algorithm for which is discussed in the next section.

The two implementations of the unconstrained network that have been disussed here clearly do not cover all possible implementations. Various special cases can be constructed which are not representataive of either. However, no other general type of implementation seems to have been identified.

III. ORDER-CONSTRAINED NETWORKS

The switching networks to be described in this section have the following properties:

a) The M output ports cannot be connected in any order to the arbitrarily selected subset M of the N beam feeds, the order being determined by the switching network.

b) the total number S of switches is usually very much less than for the unconstrained networks.

c) the maximum number L of switches in series in any signal path is no greater than for the unconstrained networks.

d) the number of possible signal paths physically in parallel is usually much smaller than for the unconstrained networks.

e) the switching algorithm is simple.

f) the general network circuit diagram evolves systematically.

Other classes of network than the one to be described, and which satisfy the same set of conditions, may exist, and there are some known special cases which are an improvement, in some respects, on the particular network of that class.

The procedure for sketching the network diagram is a process of overlaying M identical (N-M+1)-to-1 sub-trees, each one displaced laterally by the unit cell size from the one before. Figure 6 illustrates the process for an 8-to-3 network. On the left is the 6-to-1 subtree used as the building block, and on the right is the result of overlaying the necessary 3 subtrees to form the complete network. At every junction of 3 lines, a switch is required. Where lines lie on top of one another, only one line (physically, only one length of waveguide) is understood to exist.

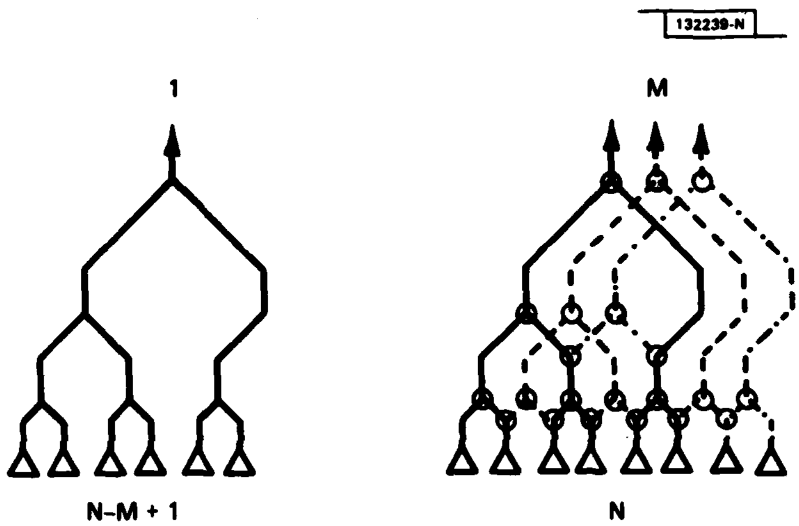

Fig. 6. Development of the complete order-constrained N-to-M network (right) by overlaying displaced replicas of an (N-M+1)-to-1 subtree (left).

The (N-M+1)-to-1 subtrees must conform to a particular style for their assembly into the complete network to proceed correctly. That style is shown, for N-M+1 running from 1 to 10, in Figure 7. Further development for larger values of N-M+1, follows in an obvious extension of the procedure. In words, the n-to-1 subtree is obtained from the complete $2^{\hat{n}}$-to-1 binary tree by removing $2^{\hat{n}}$-n ports in a block from one end, and pruning away the superfluous branches. Here $\hat{n}$ is given by $1 + \mathrm{Int}\{\log_2(n-0.1)\}$. The complete binary trees in Figure 7 are denoted by the numbers 1,2,4 and 8.

If $\log_2(N-M+1)$ is an integer, then the fact that the subtree is a complete binary tree, as discussed above, makes the resulting network symmetrical. Otherwise, it is not.

The derivation of the formula for the total number S of switches in the network is straightforward, but tedious. One way is to start with N-M=0, for which S=0, and then keep account of the increments ΔS in S which occur as N is increased by unit increments and M is kept constant. The general result is that the ΔS have a geometrically cyclic behavior, the k'th cycle extending over the 2^k values of N-M for which $2^k \leq N-M < 2^{k+1}$.

Specifically,

$$\Delta S = \begin{cases} 2M-1, \text{ for } N-M = 2^k \\ \mathrm{Min}\{2^{k-j+2}-1, 2M-1\}, \text{ for } N-M = 2^k + (2m-1)2^{k-j} \end{cases} \tag{5}$$

where $k = 0,1,2,\ldots$; $j = 1,2,3,\ldots,k$ and $m = 1,2,\ldots 2^{j-1}$. That is, when N-M is increased by unity to assume the values given above, S increases by the amount ΔS given above. Figure 8 shows the sequence N-M = 0,1,2..., 8 for the case M=3. Examining the behavior of such sequences for different values of M leads to the result expressed by Equation (5).

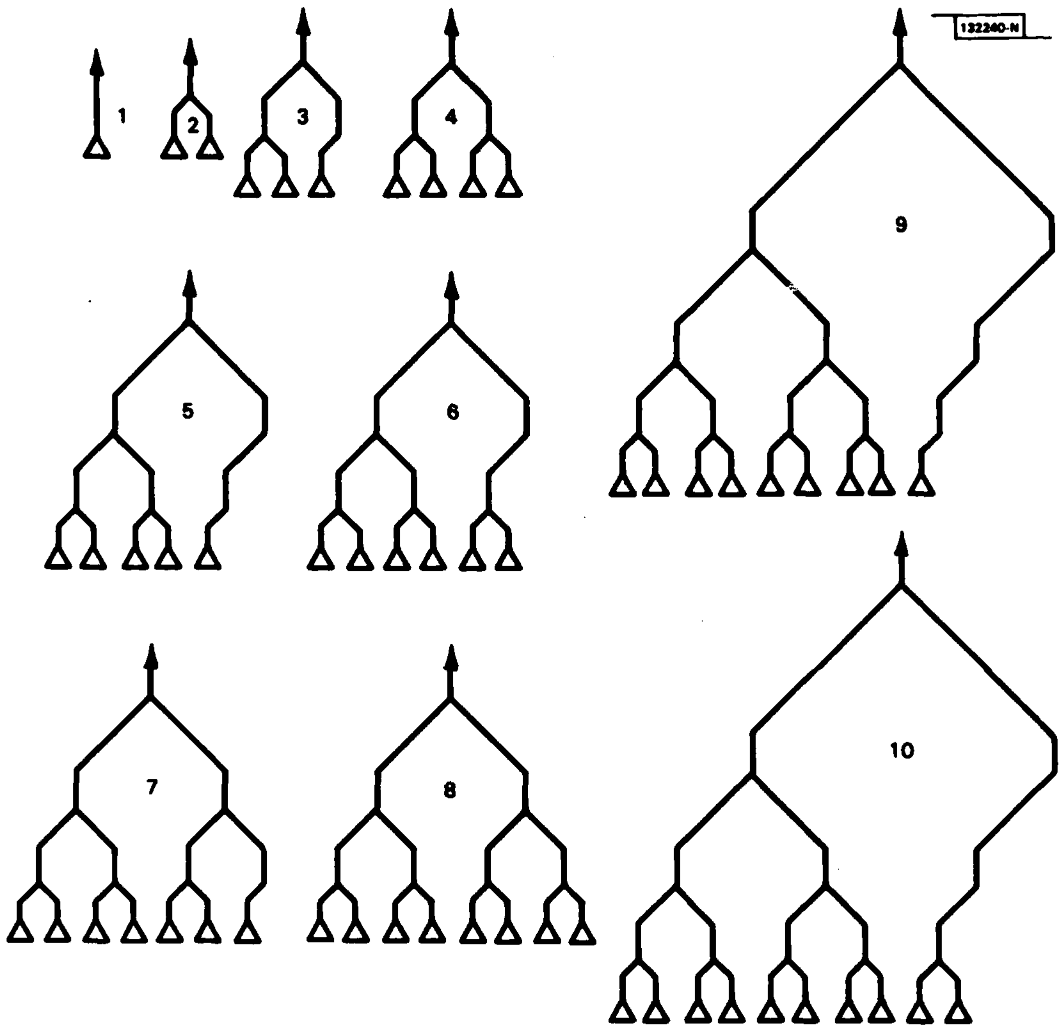

Fig. 7. Development of the basic n-to-1 subtree building block for the order-constrained N-to-M network, where n=N-M+1.

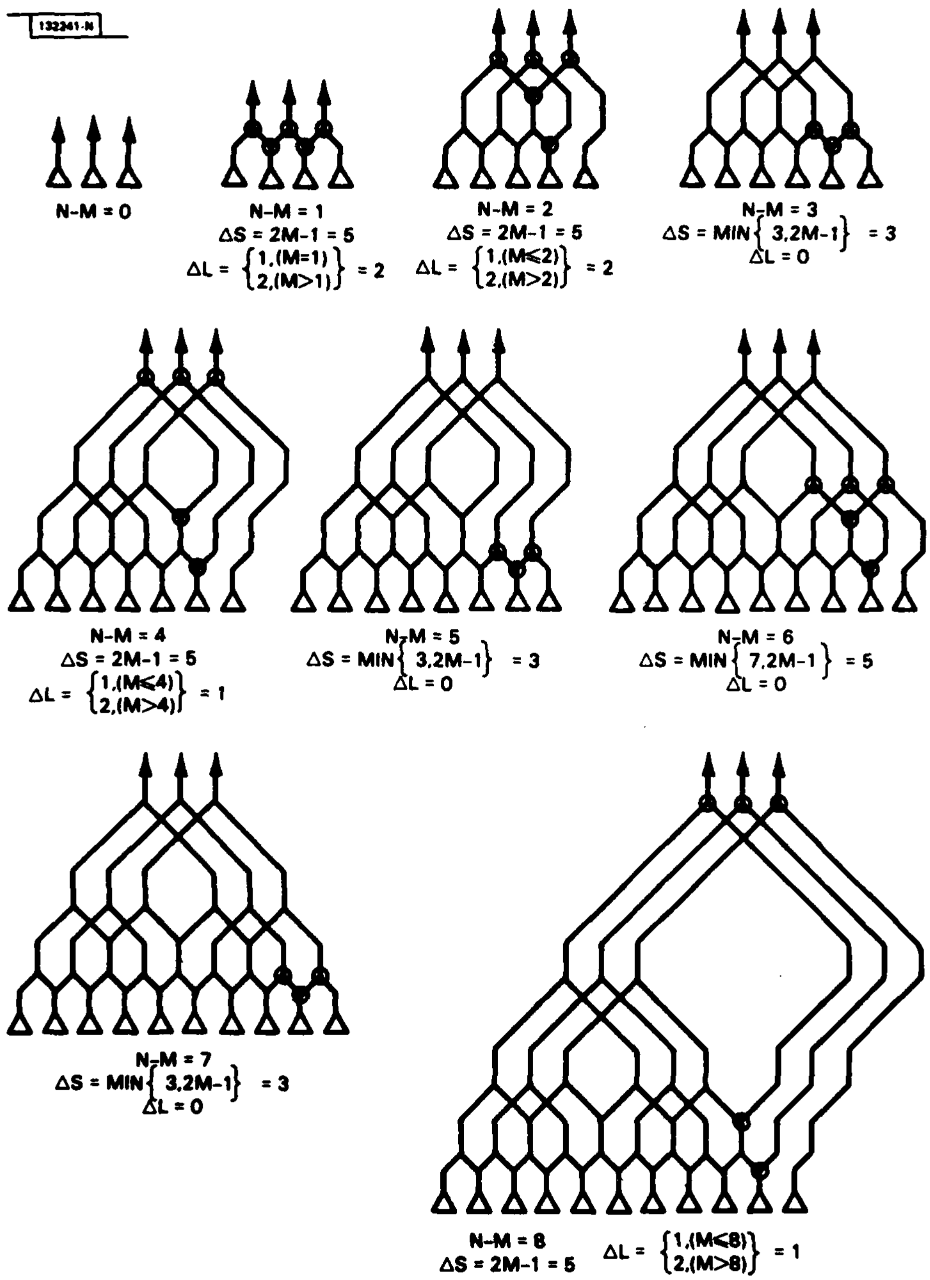

Fig. 8. Increments in S and L as N-M increases for constant M. The additional switches in each case are marked.

The result can also be depicted as the following infinite ΔS array, in which M_x stands for $\mathrm{Min}\{x,2M-1\}$:

$$\begin{array}{llllllllllllll}
(k=0) & M_\infty & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
(k=1) & M_\infty & M_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
(k=2) & M_\infty & M_3 & M_7 & M_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
(k=3) & M_\infty & M_3 & M_7 & M_3 & M_{15} & M_3 & M_7 & M_3 & 0 & 0 & 0 & 0 & 0 & \\
(k=4) & M_\infty & M_3 & M_7 & M_3 & M_{15} & M_3 & M_7 & M_3 & M_{31} & M_3 & M_7 & M_3 & M_{15} & \cdots \\
\vdots & \vdots & & & & & & & & & & & & & \vdots
\end{array}$$

The number of non-zero elements in the k'th row is the number of unit increments of N-M needed to complete the k'th cycle.

The total number of switches S required by the order-constrained N-to-M switching network is given by summing, row by row, the first N-M non-zero elements of this array. The result is

$$S = (K+1)(2M-1)+\sum_{k=1}^{K} n_k \, \mathrm{Min}\{2^{k+1}-1,2M-1\} \tag{6}$$

where $K = \mathrm{Int}\{\log_2(N-M+0.1)\}$ and $n_k = \mathrm{Max}\{0,\mathrm{Int}[(N-M+0.1)2^{-k}-1/2]\}$. The coefficient n_k is the number of times the element $\mathrm{Min}\{2^{k+1}-1,2M-1\}$ occurs in the k'th row of the ΔS array.

Following the same procedure to evaluate the maximum number L of switches in any signal path, we find the increments ΔL of L to be given by

$$\Delta L = \begin{cases} 0, & N-M \neq 2^k \\ 1, & N-M = 2^k \text{ and } M \leq 2^k \\ 2, & N-M = 2^k \text{ and } M > 2^k, \end{cases} \tag{7}$$

for $k = 0, 1, 2, \ldots$

Thus the number of non-zero increments of L is $1 + K$, where $K = \text{Int}\{\log_2(N-M+0.1)\}$, as before. And the number of increments of L of value 2 is $1+\text{Min}\{K, \text{Int}[\log_2(M-0.1)]\}$. The required expression for L is therefore

$$L = 2 + K + \text{Min}\{K, \text{Int}[\log_2(M-0.1)]\}, \tag{8}$$

where $K = \text{Int}[\log_2(N-M+0.1)]$.

The maximum number P of possible signals paths physically in parallel is the measure used to quantify the plumbing complexity of the network. It is given by

$$P = \begin{matrix} N, \; M=1 \\ 2(N-1), \; M > 1, \end{matrix} \tag{9}$$

as an examination of Figure 8 quickly verifies. When N-M is odd, P is the number of waveguides converging on the second bank of switches behind the beam feeds, in the direction of the output ports. When N-M is even, P is greater than this number by unity to include the additional path not having a switch in the first bank.

The Appendix includes tables of S, L and P, for the order-constrained network. Table I of section II shows a comparison of S, L and P, for the order-constrained network, with their values for the two classes of uncontrained network, for some selected values of N and M. It shows that the network weight, complexity and insertion loss can all be less, in many cases drastically so, if the order constraint is acceptable.

The switching algorithm for the order-constrained network is based on a simple 1-to-(N-M+1) algorithm being applied to each of the M binary subtrees separately, and then accommodating the fact that the subtrees overlap and are superimposed. (Figure 6 shows a single subtree and the superposition process.) The signal path in the complete network from beam feed n to output port m is identically the signal path from beam feed n in the subtree converging on output port m.

This definition of the signal paths (already implicit in the derivation above of the maximum number of switches in any signal path) makes explicit the ordering constraint imposed by the network. It can be expressed in terms of the list of M beam addresses $n_1, n_2, \ldots, n_m$. Beam address n_m, for example, is the number of the beam feed to which output port m is to be connected. If we label the beam feeds from 1 to N, and the output ports from 1 to M, in order, in the same sense, then the above definition of required path imposes the constraint on the n_m that

$$1 \leq n_1 < n_2 < n_3 \ldots < n_m \leq N. \tag{10}$$

This can be reexpressed as

$$m \leq n_m \leq N - M + m, \tag{11}$$

which means that output port m can be connected only to the subset of size N-M+1 of the total number of beam feeds. Figure 9 depicts this result graphically. In words, the ordering constraint means that the selected set of

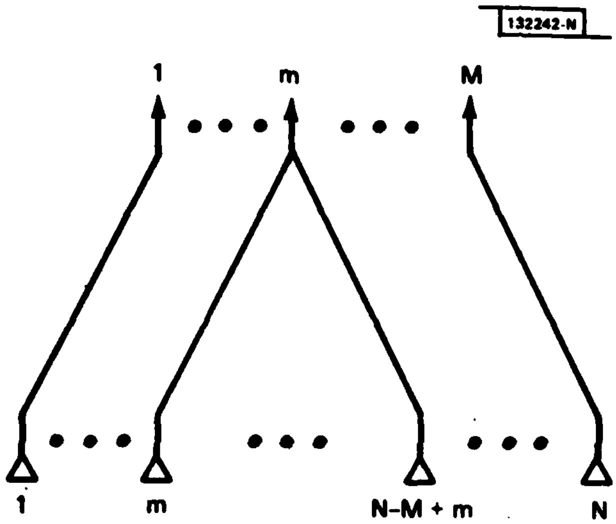

Fig. 9. Symbolic depiction of the range of beam feeds to which output port m can be connected.

m beam feeds are connected to the M output ports in the same order. The M separate simultaneous signal paths through the network do not cross one another on the planar circuit diagram. We note that the network itself does not improve the full ordering constraints described here. They are imposed by the combination of the network together with the particular definition of the signal paths adopted here.

Figure 10 shows a corner of the general order-constrained N-to-M switching network with the switches systematically numbered. There are two classes of switches, namely the T-class, which are those existing in the basic subtrees from which the network is assembled, and the U-class, which are the additional switches required at the junctions of the subtrees.

The switch settings are determined by the set of beam addresses n_m in that the individual bits $b_{r,m}$ of the binary form of the number n_m-m are precisely the settings of the T-class switches in the m'th subtree. Thus a simple procedure for setting the T-class switches is to start with m=1 and set the T-class switches according to the formula.

$$T_{r,m+kq} = b_{r,m} \tag{12}$$

where r,k = 1,2,3, ...; $q = 2^r$ and

$$\sum_{r=1}^{q} b_{r,m} 2^{r-1} = n_m - m \tag{13}$$

By repeating this procedure for successively larger values of m until m=M, we can complete the job of setting the T-class switches.

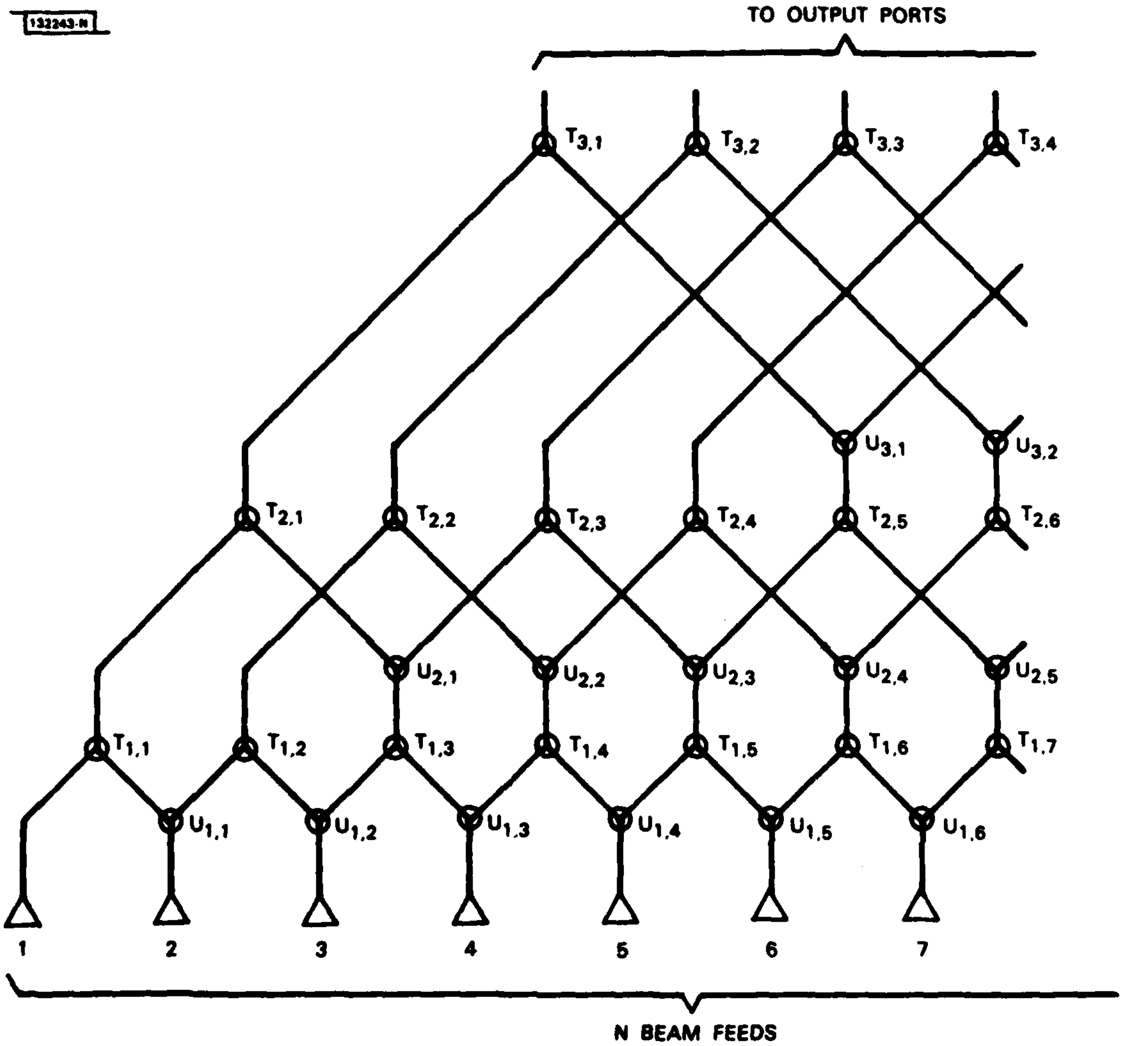

Fig. 10. Notation used to identify the switches in a switching algorithm for the order-constrained network.

However, this is costly in the expenditure of switching energy because many switches would be set and reset several times. With a little more computation, we can set just once only the switches that need to be set. The corresponding formula for this procedure is

$$T_{r,m+j} = b_{r,m} \tag{14}$$

where

$$j = \sum_{i=r+1}^{q} b_{i,m} 2^{i-1}. \tag{15}$$

Once the T-class switch settings have been determined, the U-class switch settings are given by

$$U_{r,n} = \begin{cases} 0, & T_{r,n}=1 \\ 1, & T_{r,n+p}=0 \\ \text{arbitrary otherwise} \end{cases}$$

where $p = 2^{r-1}$.

The definition of switch setting adopted here is that if $T_{r,m}$ or $U_{r,m}$ is equal to 0, the path of low insertion loss through the switch runs from a vertical connection to a connection on the left side of the switch, for the layout depicted in Figure 10. Conversely, if $T_{r,m}$ or $U_{r,m}$ is equal to 1, the side connection involved is on the right.

IV. SELECTION-CONSTRAINED NETWORKS

The unconstrained and order-constrained networks described in the previous sections have allowed the M output ports to be connected to any subset of M beam feeds selected from the total of N beam feeds. The order-constrained network obtains a substantial reduction in the switch count by restricting the order in which the selected set of M beam feeds are connected to the M output ports. This section deals with networks in which an additional constraint is imposed, namely that the selection of the M beam feeds cannot be made completely arbitrarily. It will be shown that the selection constraint can result in a further substantial reduction in the switch count and yet, in some cases, incur only a minor degradation in performance. It can do this because, of the total number of ways M beam feeds can be selected from among N, only a relatively small number are prohibited by the selection constraint.

Two types of selection-constrained switching network will be examined. These are the discrete subdivided type and the merged subdivided type. Examples are given in Figure 11, together with the corresponding order-constrained network, for the case $N = 16$, $M = 4$.

Example (b) in Figure 11 shows the most drastic kind of selection constraint in operation. Each of the M output ports can be switched only among the beam feeds in one specific subset of the N beam feeds, and each of the M subsets is discrete. It is not possible, with this network, to connect two or more outputs simultaneously to beam feeds within any single subset of the beam feeds. We can show that the traffic handling capability of such a

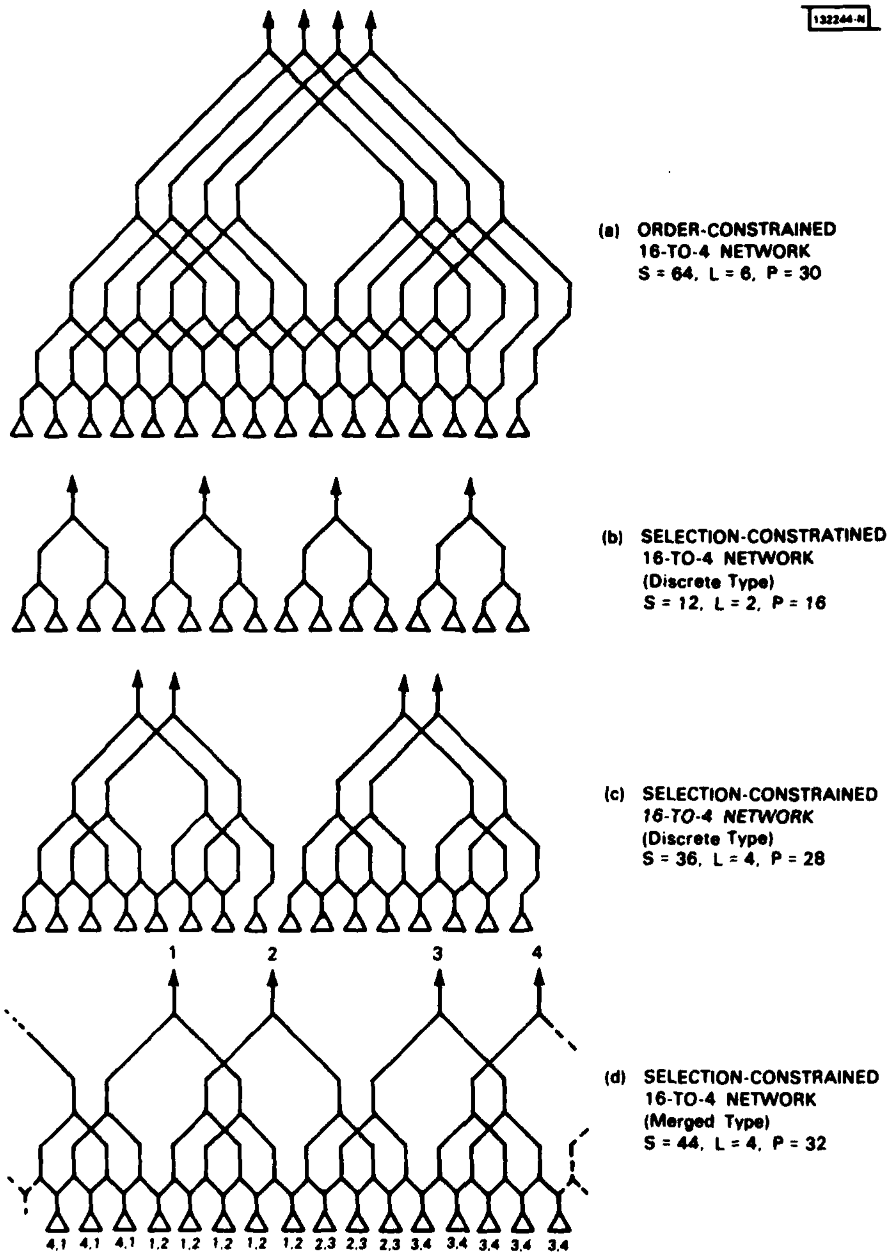

Fig. 11. Comparison of order-constrained network (a), with two examples of the discrete type of selection-constrained network (b) and (c), and with the merged type of selection-constrained network (d).

network is much inferior to that of a less constraining switching network. On the other hand, it has clearly achieved a very substantial reduction in all the measures (S, L and P) of hardware complexity.

Example (c) in Figure 11 is again a discrete type of selection-constrained network, but with a less drastic selection constraint than example (b). The reduction in hardware complexity is not as large, but it proves to have a better traffic handling capability.

To calculate S, L and P for the discrete type of selection constrained network we use the formulas and tables already developed for the order-constrained networks. This is because the network consists of two or more order-constrained networks in parallel. Thus S (the total number of switches) is the sum of the S numbers for each of the component order-constrained networks. The same applies to P (the maximum number of possible signal paths physically in parallel). On the other hand, L (the maximum number of switches in series in any signal path) is the largest of the L numbers of each of the component networks.

The merged type of selection-constrained network is shown in Figure 11(d). It is similar to the discrete type shown in Figure 11(c), but the M subtrees have been shifted laterally into a different pattern, and the circuit diagram now is imagined to be drawn around the circumference of a circular cylinder. This change has not much altered the S, L and P numbers, but the switching flexibility has been greatly improved. It will be shown to have a better traffic handling capability.

Other examples of the merged type of selection-constrained network could be given. The example shown in Figure 11(d) has an overlap of 2, in that there exists for any beam feed the possibility of connection to either of two output ports. This is because the example is generated by laterally displacing the separate subtrees of the discrete network shown in Figure 11(c), for most of whose beam feeds the same connection possibility exists. Starting with larger values of N and M, we could draw a discrete type of selection-constrained network having component order-constrained networks with three or more output ports. The corresponding merged network derived from this by laterally displacing the subtrees can accordingly have an overlap of three or more. Moreover, the merged type of selection-constrained network can have a non-uniform overlap, as shown by the example in Figure 12, an arrangement that is useful when the traffic originating in different beam footprints has different flow rates.

No general formula for evaluating S, L and P for the merged type of selection-constrained network has yet been worked out. The problem is that there is more variability in this type than any other. However, since the merged network can be derived from a discrete network, and since the derivation does not much change S, L and P, we can estimate these quantities by evaluating them for the corresponding discrete type of selection-constrained network. Such estimates are useful for system studies. Of course, for any particular network, the quantities can be obtained directly by drawing the network.

The switching algorithm for the general selection-constrained network can be applied separately to each order-constrained component, following the description of the last section, if the selection-constrained network is of the discrete type. No general formula has yet been worked out for the switching algorithm for the merged type. However, once the beam addresses for each output have been identified, and any conflicts resolved, a procedure paralleling that used for the order-constrained network can be followed.

The selection-constrained networks impede traffic flow more than the order-constrained or unconstrained networks. Since the assumption has been made here that the order constraint will be corrected, if necessary, at IF or later in the signal processing chain, it has not been necessary to compare the traffic flow properties of the unconstrained networks with the order-constrained networks. They are identical. Now, however, the traffic flow properties must be considered to enable us to trade off savings in hardware with impaired traffic flow.

A traffic flow simulation was carried out for the networks shown in Figure 11 using the following simple traffic model. The time line is divided into a chain of equal-length serving intervals separated by request slots. During each request slot, requests for service are accepted from the beam footprints and assembled in queues. Queue f stores the requests from subset f of the beam footprints. There are F queues and F separate subsets of footprints which together make up the totality of beam footprints. During each serving interval, each server (physically, a receiver fed by output port) initiates, and completes, service of the request of longest hold time in the queue(s) under the purview of that server. Requests are accepted only from "free" footprints - i.e., only from footprints not already in a request queue. Thus if queue f, which holds the requests from the m_f footprints in subset f, already holds q_f requests, then there remain only $m_f - q_f$ free footprints in that subset of footprints from which requests will be accepted. The probability that a request will originate from any particular free footprint in subset f of footprints in any particular request slot is p_f. The traffic handling capability of the network is then measured by the proportion of the time, on average, that any footprint in subset f spends waiting in its queue for service, for $f = 1, 2, \ldots, F$.

For the discrete type of selection-constrained network, each order-constrained component handles independently the traffic originating from the beam footprints it covers. Thus the performance of each component can be examined separately. In the model, therefore, there is then only one queue, the number of beam footprints is the number of feeds in the network component and the number of servers is the number of output ports in the component.

For the merged type of selection-constrained network, on the other hand, there are multiple subsets and queues. For example, for Figure 11(d), there are 4 subsets of beam footprints (containing successively 3, 5, 3 and 5 beam footprints) together with the 4 queues they feed, and 4 servers, each server having 2 queues under his purview. That is because, physically, the first 3 beam feeds can each be connected to output ports 4 or 1, the next 5 can be connected to ports 1 or 2, the next 3 to 2 or 3 and the final 5 to 3 or 4. As another example, Figure 12 shows a network which has 6 subsets of beam

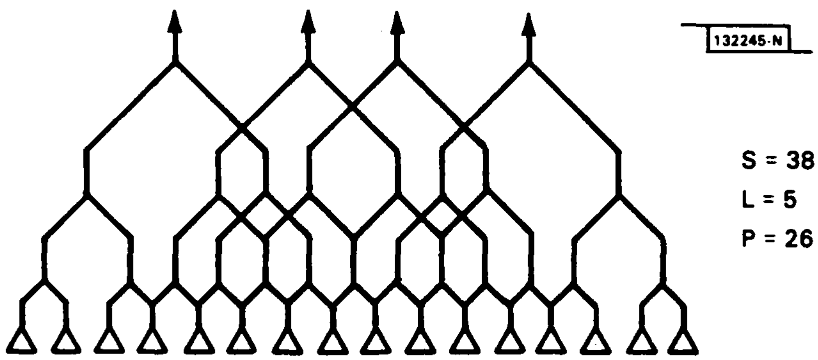

Fig. 12. An example of a selection-constrained 16-to-4 network of merged type having a non-uniform overlap.

footprints (containing, successively 3, 2, 3, 3, 2 and 3 beam footprints) together with the 6 queues they feed, and 4 servers each having purview of 3 queues.

The results obtained from the traffic-flow simulation, for the four networks shown in Figure 11, are shown graphically in Figure 13. The curves show the percentage of the time that each footprint spends waiting for service, on average, as a function of the footprint request probability p, for the different networks. The abscissa can also be interpreted as the average footprint request frequency. All footprints were assumed to have the same request frequency.

The curves in Figure 13 show that, at low values of the request frequency, the waiting time percentage is very different for the four networks, but its absolute value is small. Thus the simplest switching network (case (b) in Figure 11) incurs 20 times the waiting time percentage of the most complicated network (case (a) in Figure 11), when $p = 0.1$, but all the waiting time percentages are 2% or less. As the request frequency increases, the waiting time percentages increase for all networks, but they also get closer and closer to being equal. For large request frequencies, all waiting time percentages approach the value 75%. This is because then all footprints spend virtually all their time waiting in the queue (or queues), and since there are 4 times as many footprints as servers (output ports), each footprint is served on average 25% of the time, and spends the rest of the time waiting while others are served.

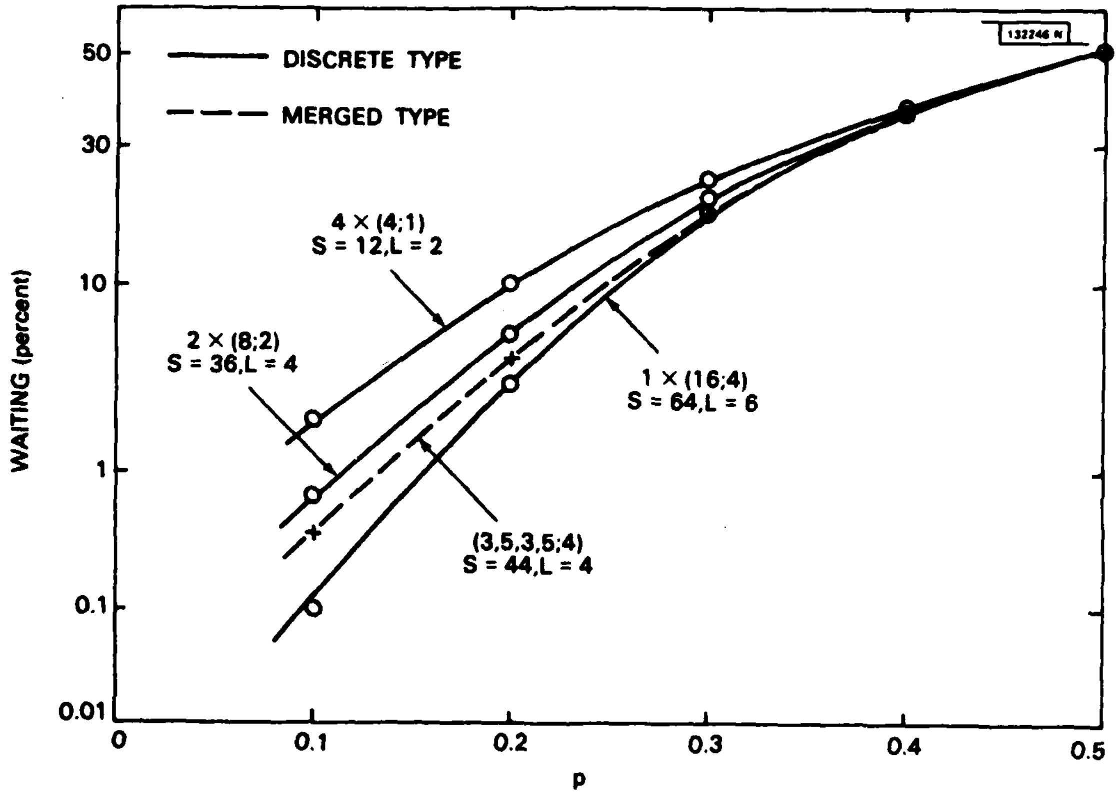

Fig. 13. The performance of the four networks of Figure 11, for uniform request frequencies. The notation Kx(N;M) implies a discrete type of selection constrained network with K components, each switching M ports among N feeds.

By considering such sets of curves for a range of values of M, we can pick the network which best fits the system requirements. In doing so, we would have to consider the weight and power tradeoff between switching network complexity, on the one hand, and the use of multiple receivers, on the other. If, for example, switches are relatively heavy, a requirement for maximum waiting time percentage may more economically be met with the simplest discrete type of order-constrained network with 4 output ports and 4 receivers instead of a more complicated network having fewer output ports and receivers.

Other considerations, such as providing redundancy and the effect of non-uniform request frequencies, also would be included in the choice of the optimum network. In this context, it should be noted that the merged type of selection-constrained network can accommodate non-uniform traffic conditions much better than the discrete type.

Table II compares the waiting time percentage for the networks shown in Figure 11(c) and 11(d) when the request frequency for the footprints covered by the first 8 beam feeds is 0.3, and is 0.1 for the remaining 8. For both networks, the waiting time percentage is larger for the footprints in the subset having the greater request frquency. However, this waiting time for the discrete network, which cannot recruit the underworked receivers connected to the second set of 8 feeds to help in serving the busier 8, is twice that of the merged network. For the same reason, the waiting time percentage of the discrete network for the less busy set of feeds is only half that of the merged network. Thus the ability of the merged type of network to shift the

TABLE II

WAITING TIME PERCENTAGES OF NETWORKS OF FIGURES 11(c) AND 11(d)

			Waiting (%)	
f	m_f	p_f	Discrete	Merged
1	8	0.3	21.0	9.5
2	8	0.1	0.7	1.2

TABLE III

NETWORK OF FIGURE 11(d) WITH TWO SERVING STRATEGIES

			Waiting Time per Footprint		Holding Time per Request		Length of Queue	
f	m_f	p_f	'Oldest 1st'	'Longest 1st'	'Oldest 1st'	'Longest 1st	'Oldest 1st'	'Longest 1st'
1	3	0.2	14.0	31.4	0.81	2.29	0.42	0.94
2	5	0.4	26.8	23.4	0.92	0.76	1.34	1.17
3	3	0.2	15.2	31.1	0.90	2.26	0.46	0.93
4	5	0.4	25.3	21.5	0.85	0.68	1.27	1.08

servers laterally can bring more service to the more heavily loaded feeds, wherever they may be. This tends to equalize the waiting time percentages experienced by footprints having different request frequencies.

Of course, the serving strategy is important here, too. The one described above, and used to obtain the dashed curve in Figure 13, specified that each receiver serves first the request of longest holding time in the queues under its purview. An alternative strategy is that each receiver serves first the request at the head of the longest queue under the purview of that receiver. This seems, on the face of it, a reasonable procedure. And simulations show that essentially the same curve as that shown dashed in Figure 13 would apply when this alternative strategy is used on the same merged type of selection-constrained network.

Under conditions of strongly non-uniform traffic load, however, the two strategies lead to important differences in performance. In particular, the "oldest-request-first" strategy tends to equalize the average length of time that the individual requests are held waiting in queues, whereas the "longest-line-first" strategy tends to equalize the average length of the queues. If, therefore, one queue is supplied from a small subset of footprints whereas the next is supplied from a large subset, then when the request frequency from the large subset is large enough, the corresponding queue will almost always be longer than the other queue, and therefore will essentially capture the receiver using the "longest-line-first" strategy. The result is that the requests originating from the smaller subset of footprints then spend much longer waiting in their slowly moving, short, queue than do the requests in the longer but quickly moving queue. The serving strategy of "oldest-request-first" type, however, prevents this.

Table III shows some simulation results obtained with the 4-server, 4-queue network shown in Figure 11(d). The four subsets of footprints contain, respectively, 3, 5, 3 and 5 footprints, and the corresponding request frequencies for each free footprint in the subsets are 0.2, 0.4, 0.2 and 0.4. Thus two of the subsets each generate a maximum total request frequency of 0.6 requests per serving interval, and the other two generate a maximum total of 2.0 requests per serving interval. The percentage of the time each footprint in the subsets stands waiting for service (the waiting percentage, w_f) is given in columns 4 and 5. For column 4, each server used the "oldest-request first" strategy, and for column 5, the "longest-queue-first" strategy, to pick from the two queues under his purview the next footprint to receive service. The corresponding average time t_f, in serving intervals, that each request spends standing in its queue, is given in columns 6 and 7, and the average length l_f, in requests, of the queues is given in columns 8 and 9. (The relationships between w_f, t_f and l_f are $l_f = m_f w_f$ and $t_f = w_f/[p_f(p_f(1-w_f)]$.)

The results in Table III make clear the differences between the two strategies. The length of queue is held substantially uniform by the "longest-queue-first" strategy, and holding time per request is held substantially uniform by the "oldest-request-first" strategy. At the same time, both performance measures show a three-to-one variation for the strategies in the reverse order, a variation which matches the variation in the offered traffic load. In particular, the capture phenomenon, in which the busier subsets deny service to the less busy subsets, when the "longest-queue-first" strategy is used, is clearly demonstrated in column 7.

The 16-to-4 selection-constrained networks shown in Figures 11 and 12, while useful because of their simplicity in demonstrating the properties of such networks, are not representative of the more elaborate beam-switching systems of current interest. Accordingly, the subjects of the next few figures are various selection-constrained 64-to-M networks, where M takes the values 4, 8 and 16.

For each value of M, various selection-constrained networks of both the discrete and merged type are included. Figures 14 and 15 cover four such networks for which M=4. Using the nomenclature of Figure 13, these networks are the two discrete types defined parametrically as 4x(16;1) and 2x(32;2), together with the limiting case 1x(64;4), and also the merged type defined as (17, 15, 17, 15;4), in which each server has two queues within its purview. The circuit diagrams of the discrete type of network are self evident. That for the merged type is shown, in part, in Figure 14. The circuit is imagined to be laid out on the surface of a cylinder and is symmetrical about the sections A and B, shown, as well as two other similar sections not shown.

The performance of the four networks is shown by the curves in Figure 15. Again we conclude that the merged type of selection-constrained network does not quite meet the performance of the order-constrained or unconstrained networks, but its hardware complexity is substantially less. On the other hand, it does better than the discrete type of selection-constrained

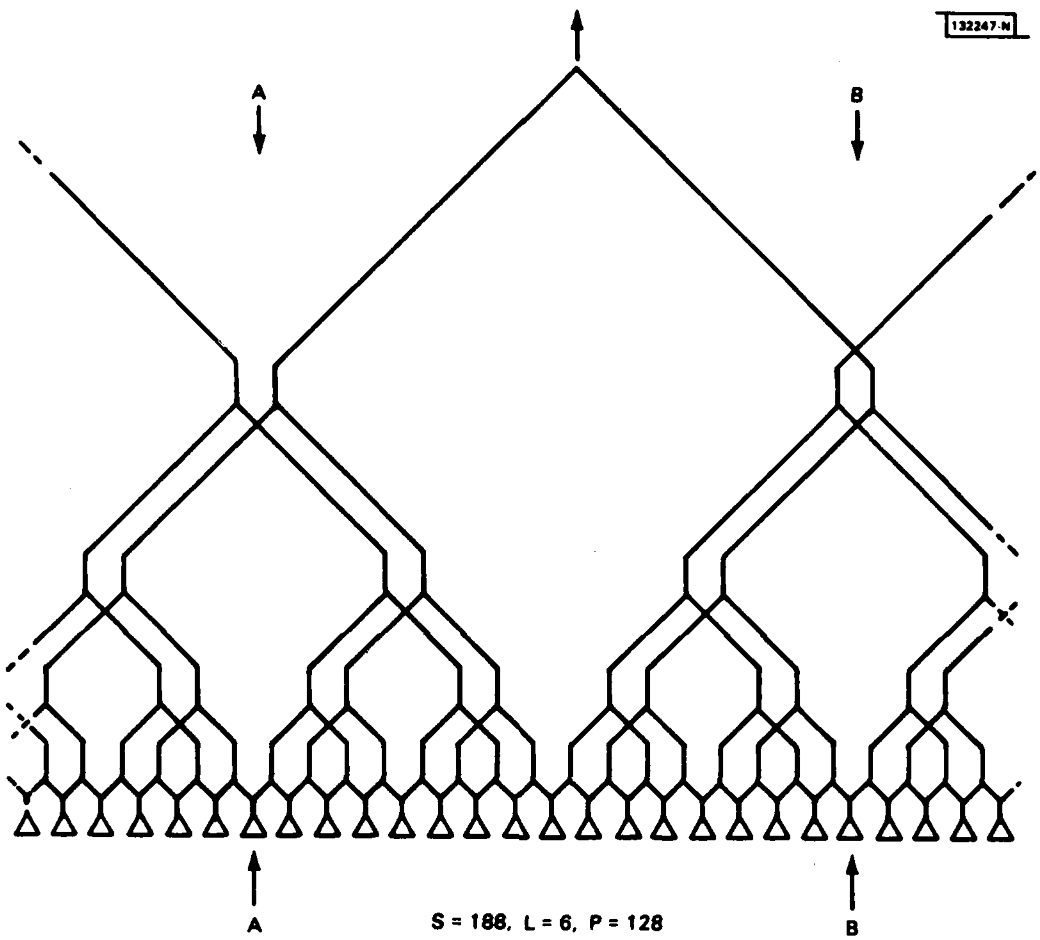

Fig. 14. Selection-constrained 64-to-4 network of merged type. Only one complete cell is shown of the four needed to complete the network.

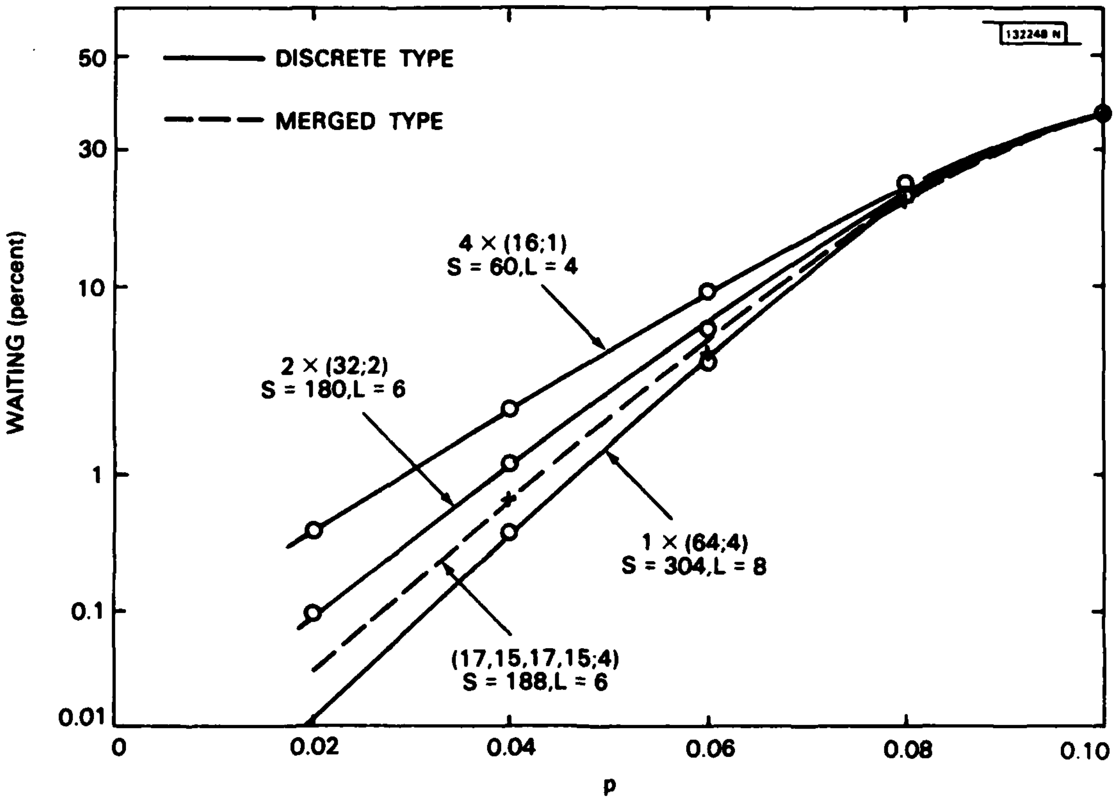

Fig. 15. The performance of four 64-to-4 selection-constrained networks for uniform request frequencies.

network having essentially the same hardware complexity. Moreover, based on the results shown in Table II for the 16-to-4 networks, we know the merged type of network to perform better than the discrete type when the traffic load is non-uniform.

For the 64-to-8 networks, we consider three discrete types, namely 8x(8;1), 4x(16;2) and 2x(32;4), together with the limiting case 1x(64;8), and two merged types, namely (9,7,9,7,9,7,9,7;8) and (10,7,10,5,10,7,10,5,;8). There are two queues under the purview of each server in the first of these merged networks, and four in the second network. As before, the circuit diagrams are self evident for the discrete type of network. For the merged type, the first one is sketched in Figure 16, and the second is readily derived from the 64-to-4 network shown in Figure 14 by superimposing on the network of Figure 14 a second identical network shifted laterally by 10 feed positions.

The curves of Figure 17 summarize the performance of these networks. Much the same conclusions apply here as to the 64-to-4 network results, except that now there is a new feature - the additional merged network in which each server has four queues under his purview. This particular merged type of selection-constrained network has a performance essentially indistinguishable from that of the order-constrained network, and yet it has 25% fewer switches and 2 fewer switches in series in its signal paths.

Figure 17 further emphasizes the improvement in performance that can be obtained from a discrete type of network by simply shifting its basic subtrees laterally to overlap one another in a more or less uniformly distributed

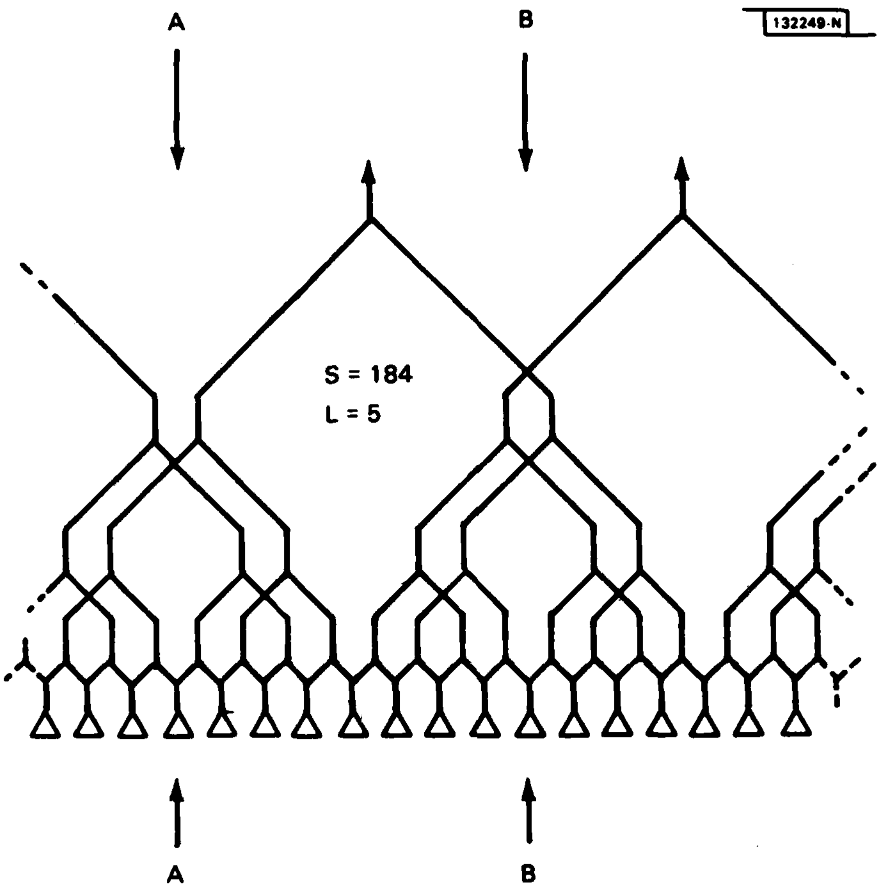

Fig. 16. Selection-constrained 64-to-8 network of merged type. Only one complete cell is shown of the eight needed for the complete network. There are a total of eight sections of symmetry like the two shown at A and B.

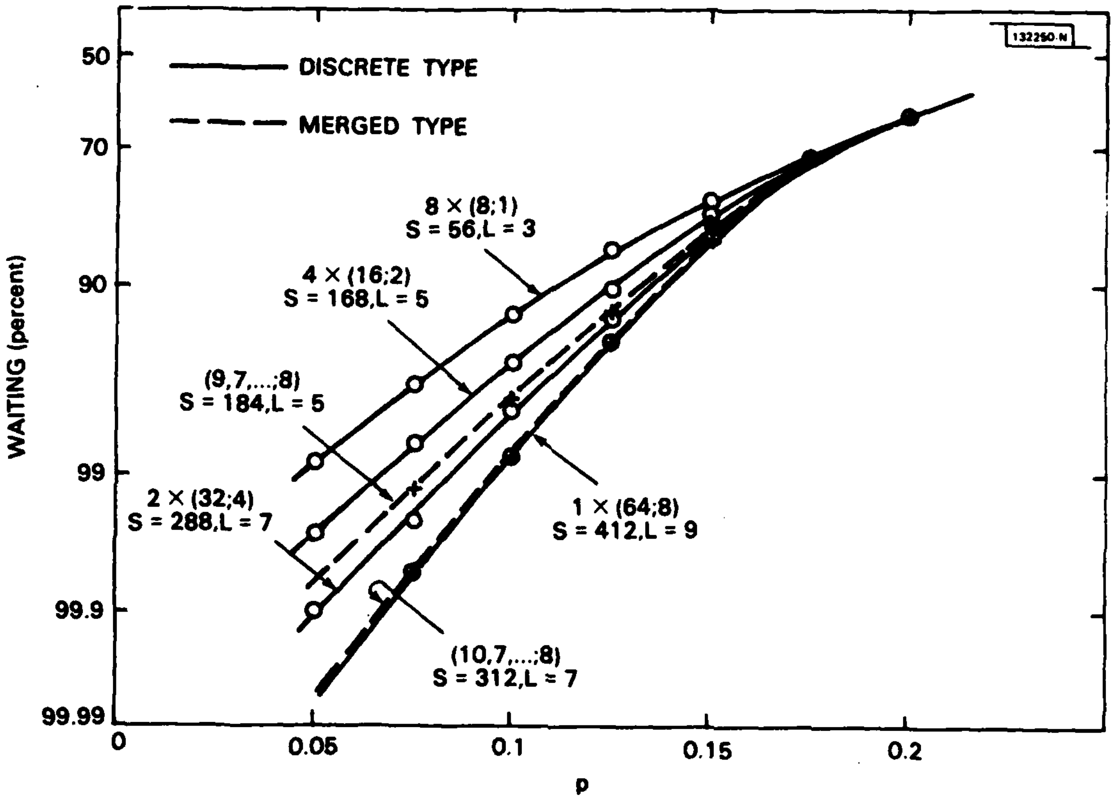

Fig. 17. The performance of six 64-to-8 selection-constrained networks for uniform request frequencies.

manner. Shifting the subtrees of the discrete 4x(16;2) network produced the merged (9,7,...;8) network. Similarly, the 2x(32;4) network was transformed into the merged (10,7,...;8) network.

Figures 18 and 19 cover the 64-to-16 selection-constrained networks, including four of the discrete type, namely 16x(4;1), 8x(8;2), 4x(16;4) and 2x(32;8), together with the limiting case 1x(64;16), and two of the merged type, namely (5,3,5,3,...;16), in which each server has two queues under his purview, and (7,2,5,2,...;16), in which each server has four. The circuit diagrams of the discrete type are self evident, since each one is a number of separate order-constrained networks. The merged type is developed as before by displacing the basic subtrees of the corresponding discrete type. In fact, the notation (n_1, n_2, n_3, n_4...;M), in which the sequence n_1, n_2,...contains M members, and whose sum equals N, also specifies the precise circuit diagram, provided we also know the number J of queues each server has under his purview, and provided that J is the same for every server. For then, there are M subtrees, M/J of which contain a total of N feeds (so that M of them can cover N feeds J times) and therefore each subtree is an [N/(M/J)]-to-1 subtree. By drawing M such subtrees, each one laterally displaced from the previous one successively by n_1, n_2,..., the complete circuit diagram is obtained. It is clear that one more requirement for the validity of this procedure is that both M/J and N/(M/J) must be integers.

The circuit of the network (5,3,5,3,...;16) is shown in Figure 18, in part. The complete circuit is the logical continuation of the part shown around the surface of a cylinder until 16 output ports are obtained. The

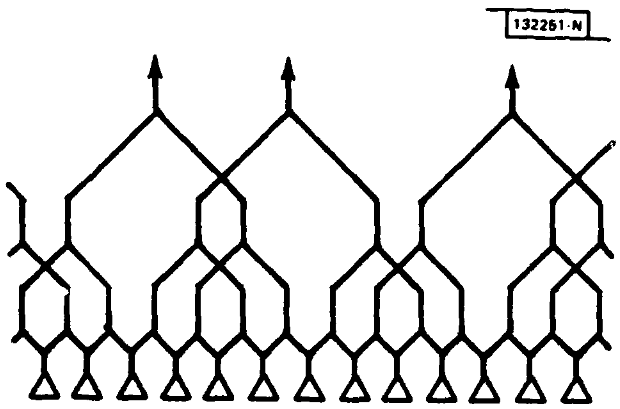

Fig. 18. The basic structure of the merged 64-to-16 network of the form (5,3,5,3,...;16).

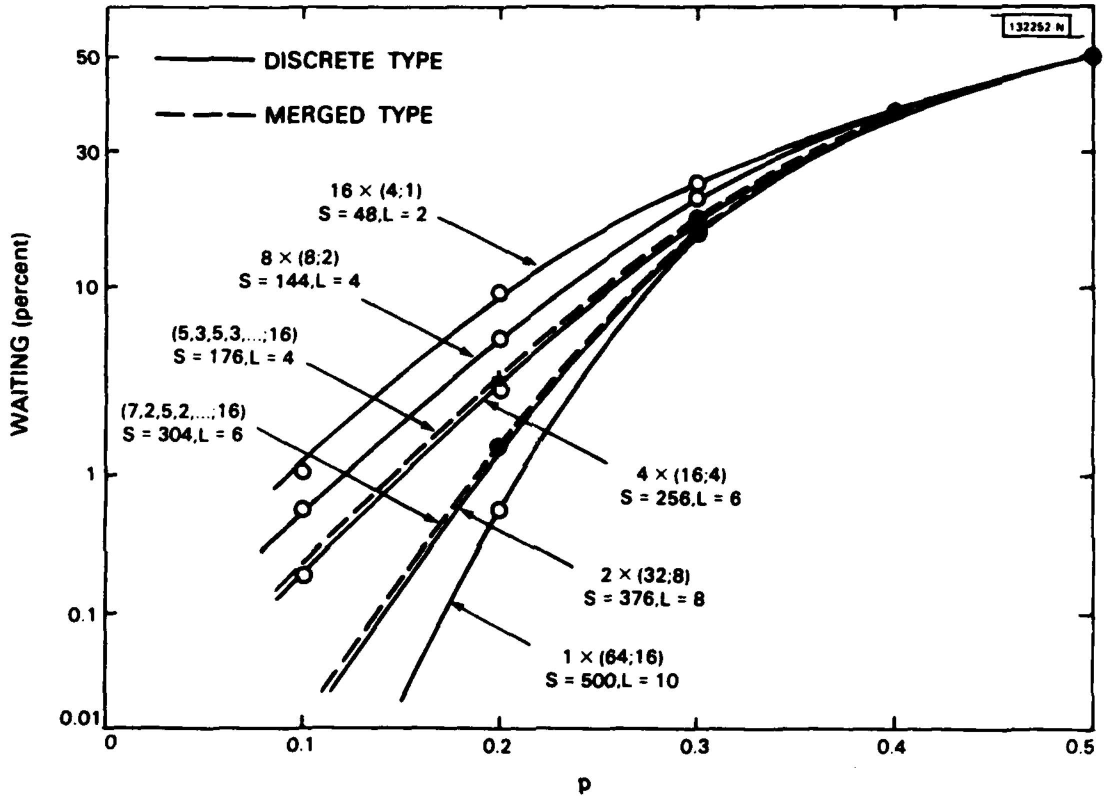

Fig. 19. The performance of seven 64-to-16 selection-constrained networks for uniform request frequencies.

network (7,2,5,2,...;16) is not shown because it is readily deduced from the circuit shown in Figure 16 by overlaying a second complete identical 64-to-8 network on top of the original, but displaced laterally by two feed spacings.

The simulation results shown by the curves in Figure 19 are consistent with the earlier results. Again, the operation of converting a discrete type of selection-constrained network to a merged type by moving its subtrees laterally produces a marked improvement in its performance without much changing its hardware complexity.

It should be noted that one of the merged type of networks derivable by this technique is missing from the list of those 64-to-16 networks studied. It is the one obtained from the 2x(32,8) discrete network. The omission is deliberate. Its complexity would seem to rule it out as a practical possibility. We would expect its performance to be essentially indistinguishable from that of the 1x(64;16) network. This latter network <u>is</u> included, however, because it represents the upper bound on the performance of all 64-to-16 networks. It is the reference performance for judging the others.

A telling demonstration of the benefits of using the merged type of network rather than the discrete is presented in Figure 20. Three curves are plotted showing the variation of the waiting percentage per footprint as the number of switches in the network varies for a constant value of request frequency. Only the discrete type of network is included in the points defining the curves. The merged networks are plotted as separate points. It is clear from their position that they provide a distinctly better tradeoff between hardware complexity and performance than do the discrete type.

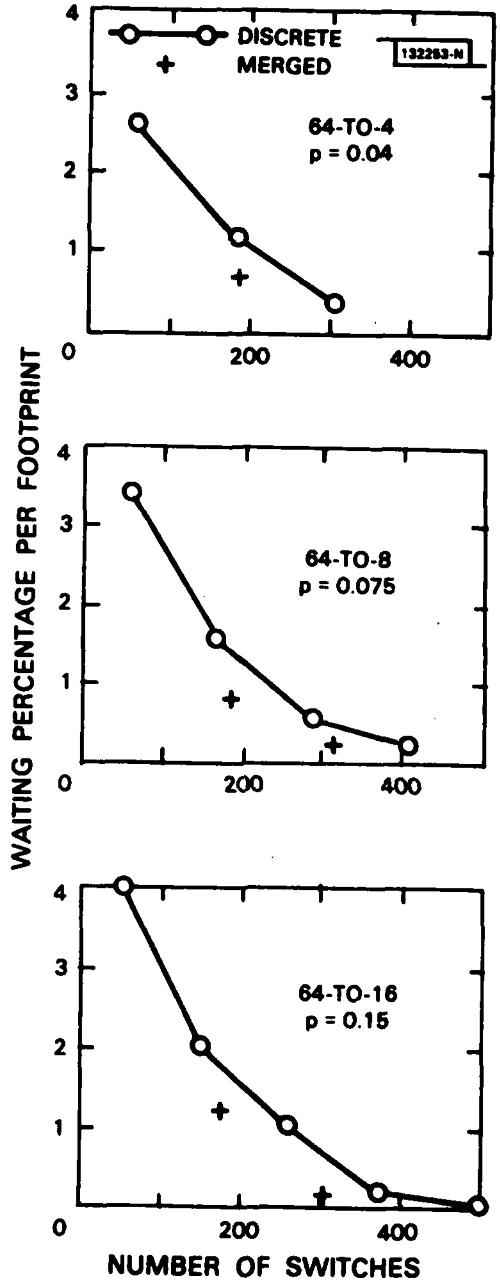

Fig. 20. Trade-off of complexity and performance for discrete and merged types of network.

It has been mentioned earlier in this section that the switching algorithm for the merged type of network has yet to be defined in a systematic way. It is necessary to note that there is also a restriction in the physical connectivity of the merged type of network when the servers each have three or more queues under their purview. After applying any appropriate serving strategy to obtain the list of footprints next to receive service we find, under rare circumstances, that no matter how we try to allocate the footprints to be served among the servers, we cannot connect two adjacent feeds simultaneously to two servers, even though either feed alone can be connected to either of the servers.

The problem is illustrated in Figure 21. When there are more than two queues under the purview of each server, we find that at some places in the network, *an additional switch* is necessary (shown at point P), immediately following the first two in the signal flow path, to determine at which of two outputs ports the signal will eventually emerge. If our serving strategy dictates that feeds A and B should be connected to ports X and Y, we find ourselves incapable of complying, because to get to ports X and Y, the signals from feeds A and B both have to pass through the additional switch.

Fortunately, such a serving requirement is met only with extreme rarity and so it does not invalidate the simulation results for the more complicated merged networks. The event is rare because, if there are three or more queues under the purview of each server, the footprints to be served can usually be reallocated among the servers to avoid this conflict. However, it does complicate the development of a switching algorithm.

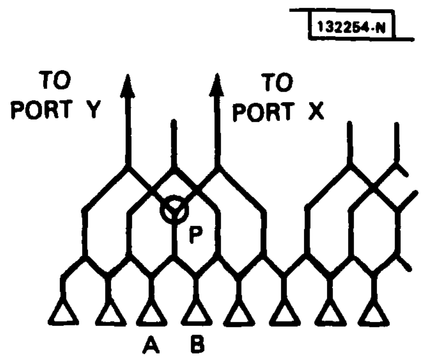

Fig. 21. Position of network illustrating a possible connectivity problem exhibited by merged networks.

V. CONCLUSIONS

The extreme hardware complexity of unconstrained N-to-M switching circuits is to be avoided whenever possible. Accepting an ordering constraint in the way the M selected feeds are connected to the M output ports allows a substantial reduction in this complexity. This class of order-constrained network can be described in a general theory which determines a specific circuit diagram and a specific simple switching algorithm. The next step in reduced complexity is the merged type of selection-constrained network, which physically does not provide for some of the conceivable ways of selecting any M from N. This class of network can encompass much greater variability than the order-constrained class. As a result, there are guidelines for the design of such circuits, but no complete theory.

The results of traffic flow simulations through the various networks demonstrate a performance of the merged type of selection-constrained network that makes it a strong candidate in the list of networks to be considered in practice.

VI. APPENDIX

This Appendix contains tables of S (the total number of switches in the network), L (the maximum number of switches in series in any signal path) and P (the maximum number of possible signal paths physically in parallel) for the unconstrained switching networks of both types, and the order-constrained networks. The three numbers, together, characterize the hardware complexity of the network, as well as (in the case of L) being an indication of the insertion loss.

For the directly implemented type of unconstrained network, S, L and P are given by

$$S_u(N,M) = 2NM - N - M$$
$$L_u(N,M) = 2+\mathrm{Int}\{\log_2(N-0.1)\} + \mathrm{Int}\{\log_2(M-0.1)\}$$
$$P_u(N,M) = NM.$$

These are tabulated in Table IV for a range of values of N and M. The general circuit of this network is shown in Figure 1.

For the compound implementation of the unconstrained network, S, L and P are given by

$$S = S_u(M,M) + S_c(N,M)$$
$$L = L_u(M,M) + L_c(N,M)$$
$$P = \mathrm{Max}\ \{M^2,\ P_c\}.$$

where the suffixes refer to the formulas for the unconstrained or order-constrained networks. These are tabulated in Table V. The general circuit of this network is shown in Figure 4.

For the order-constrained network, S, L and P are given by

$$S_c(N,M) = (K+1)(2M-1) + \sum_{k=1}^{K} \mathrm{Max}\{0,\mathrm{Int}[(N-M+0.1)2^{-k}-0.5]\}\ \mathrm{Min}\{2^{k+1}-1,2M-1\}$$

$$L_c(N,M) = 2 + K + \mathrm{Min}\{K,\mathrm{Int}[\log_2(M-0.1)]\}$$

$$\text{where } K = \mathrm{Int}\{\log_2(N-M+0.1)\}$$

$$P_c(N,M) = \begin{cases} N, & M = 1 \\ 2(N-1), & M > 1. \end{cases}$$

These are tabulated in Table VI. The development of the circuit diagram is illustrated in Figures 6 and 7.

TABLE IVa

TOTAL NUMBER S OF SWITCHES FOR UNCONSTRAINED N-TO-M NETWORK (DIRECT IMPLEMENTATION ($N \geq M$)

N \ M=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	2	7	0	-	-	-	-	-	-	-	-	-	-	-	-	-
4	3	10	17	0	-	-	-	-	-	-	-	-	-	-	-	-
5	4	13	22	31	0	-	-	-	-	-	-	-	-	-	-	-
6	5	16	27	38	49	0	-	-	-	-	-	-	-	-	-	-
7	6	19	32	45	58	71	0	-	-	-	-	-	-	-	-	-
8	7	22	37	52	67	82	97	0	-	-	-	-	-	-	-	-
9	8	25	42	59	76	93	110	127	0	-	-	-	-	-	-	-
10	9	28	47	66	85	104	123	142	161	0	-	-	-	-	-	-
11	10	31	52	73	94	115	136	157	178	199	0	-	-	-	-	-
12	11	34	57	80	103	126	149	172	195	218	241	0	-	-	-	-
13	12	37	62	87	112	137	162	187	212	237	262	287	0	-	-	-
14	13	40	67	94	121	148	175	202	229	256	283	310	337	0	-	-
15	14	43	72	101	130	159	188	217	246	275	304	333	362	391	0	-
16	15	46	77	108	139	170	201	232	263	294	325	356	387	418	449	0
17	16	49	82	115	148	181	214	247	280	313	346	379	412	445	478	511
18	17	52	87	122	157	192	227	262	297	332	367	402	437	472	507	542
19	18	55	92	129	166	203	240	277	314	351	388	425	462	499	536	573
20	19	58	97	136	175	214	253	292	331	370	409	448	487	526	565	604
21	20	61	102	143	184	225	266	307	348	389	430	471	512	553	594	635
22	21	64	107	150	193	236	279	322	365	408	451	494	537	580	623	666
23	22	67	112	157	202	247	292	337	382	427	472	517	562	607	652	697
24	23	70	117	164	211	258	305	352	399	446	493	540	587	634	681	728
25	24	73	122	171	220	269	318	367	416	465	514	563	612	661	710	759
26	25	76	127	178	229	280	331	382	433	484	535	586	637	688	739	790
27	26	79	132	185	238	291	344	397	450	503	556	609	662	715	768	821
28	27	82	137	192	247	302	357	412	467	522	577	632	687	742	797	852
29	28	85	142	199	256	313	370	427	484	541	598	655	712	769	826	883
30	29	88	147	206	265	324	383	442	501	560	619	678	737	796	855	914
31	30	91	152	213	274	335	396	457	518	579	640	701	762	823	884	945
32	31	94	157	220	283	346	409	472	535	598	661	724	787	850	913	976

TABLE IVa (cont'd)

33	32	97	162	227	292	357	422	487	552	617	682	747	812	877	942	1007
34	33	100	167	234	301	368	435	502	569	636	703	770	837	904	971	1038
35	34	103	172	241	310	379	448	517	586	655	724	793	862	931	1000	1069
36	35	106	177	248	319	390	461	532	603	674	745	816	887	958	1029	1100
37	36	109	182	255	328	401	474	547	620	693	766	839	912	985	1058	1131
38	37	112	187	262	337	412	487	562	637	712	787	862	937	1012	1087	1162
39	38	115	192	269	346	423	500	577	654	731	808	885	962	1039	1116	1193
40	39	118	197	276	355	434	513	592	671	750	829	908	987	1066	1145	1224
41	40	121	202	283	364	445	526	607	688	769	850	931	1012	1093	1174	1255
42	41	124	207	290	373	456	539	622	705	788	871	954	1037	1120	1203	1286
43	42	127	212	297	382	467	552	637	722	807	892	977	1062	1147	1232	1317
44	43	130	217	304	391	478	565	652	739	826	913	1000	1087	1174	1261	1348
45	44	133	222	311	400	489	578	667	756	845	934	1023	1112	1201	1290	1379
46	45	136	227	318	409	500	591	682	773	864	955	1046	1137	1228	1319	1410
47	46	139	232	325	418	511	604	697	790	883	976	1069	1162	1255	1348	1441
48	47	142	237	332	427	522	617	712	807	902	997	1092	1187	1282	1377	1472
49	48	145	242	339	436	533	630	727	824	921	1018	1115	1212	1309	1406	1503
50	49	148	247	346	445	544	643	742	841	940	1039	1138	1237	1336	1435	1534
51	50	151	252	353	454	555	656	757	858	959	1060	1161	1262	1363	1464	1565
52	51	154	257	360	463	566	669	772	875	978	1081	1184	1287	1390	1493	1596
53	52	157	262	367	472	577	682	787	892	997	1102	1207	1312	1417	1522	1627
54	53	160	267	374	481	588	695	802	909	1016	1123	1230	1337	1444	1551	1658
55	54	163	272	381	490	599	708	817	926	1035	1144	1253	1362	1471	1580	1689
56	55	166	277	388	499	610	721	832	943	1054	1165	1276	1387	1498	1609	1720
57	56	169	282	395	508	621	734	847	960	1073	1186	1299	1412	1525	1638	1751
58	57	172	287	402	517	632	747	862	977	1092	1207	1322	1437	1552	1667	1782
59	58	175	292	409	526	643	760	877	994	1111	1228	1345	1462	1579	1696	1813
60	59	178	297	416	535	654	773	892	1011	1130	1249	1368	1487	1606	1725	1844
61	60	181	302	423	544	665	786	907	1028	1149	1270	1391	1512	1633	1754	1875
62	61	184	307	430	553	676	799	922	1045	1168	1291	1414	1537	1660	1783	1906
63	62	187	312	437	562	687	812	937	1062	1187	1312	1437	1562	1687	1812	1937
64	63	190	317	444	571	698	825	952	1079	1206	1333	1460	1587	1714	1841	1968

TABLE IVb

MAXIMUM NUMBER L OF SWITCHES IN SIGNAL PATH FOR UNCONSTRAINED N-TO-M NETWORK (DIRECT IMPLEMENTATION) (N $\geq$ M)

N \ M=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	2	3	0	-	-	-	-	-	-	-	-	-	-	-	-	-
4	2	3	4	0	-	-	-	-	-	-	-	-	-	-	-	-
5	3	4	5	5	0	-	-	-	-	-	-	-	-	-	-	-
6	3	4	5	5	6	0	-	-	-	-	-	-	-	-	-	-
7	3	4	5	5	6	6	0	-	-	-	-	-	-	-	-	-
8	3	4	5	5	6	6	6	0	-	-	-	-	-	-	-	-
9	4	5	6	6	7	7	7	7	0	-	-	-	-	-	-	-
10	4	5	6	6	7	7	7	7	8	0	-	-	-	-	-	-
11	4	5	6	6	7	7	7	7	8	8	0	-	-	-	-	-
12	4	5	6	6	7	7	7	7	8	8	8	0	-	-	-	-
13	4	5	6	6	7	7	7	7	8	8	8	8	0	-	-	-
14	4	5	6	6	7	7	7	7	8	8	8	8	8	0	-	-
15	4	5	6	6	7	7	7	7	8	8	8	8	8	8	0	-
16	4	5	6	6	7	7	7	7	8	8	8	8	8	8	8	0
17	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
18	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
19	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
20	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
21	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
22	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
23	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
24	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
25	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
26	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
27	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
28	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
29	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
30	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
31	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
32	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9

TABLE IVb (cont'd)

33	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
34	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
35	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
36	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
37	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
38	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
39	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
40	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
41	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
42	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
43	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
44	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
45	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
46	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
47	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
48	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
49	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
50	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
51	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
52	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
53	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
54	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
55	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
56	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
57	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
58	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
59	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
60	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
61	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
62	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
63	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
64	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10

TABLE IVc

MAXIMUM NUMBER P OF PHYSICAL SIGNAL PATHS FOR UNCONSTRAINED N-TO-M NETWORK (DIRECT IMPLEMENTATION) ($N \geq M$)

N \ M=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	2	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	3	6	0	-	-	-	-	-	-	-	-	-	-	-	-	-
4	4	8	12	0	-	-	-	-	-	-	-	-	-	-	-	-
5	5	10	15	20	0	-	-	-	-	-	-	-	-	-	-	-
6	6	12	18	24	30	0	-	-	-	-	-	-	-	-	-	-
7	7	14	21	28	35	42	0	-	-	-	-	-	-	-	-	-
8	8	16	24	32	40	48	56	0	-	-	-	-	-	-	-	-
9	9	18	27	36	45	54	63	72	0	-	-	-	-	-	-	-
10	10	20	30	40	50	60	70	80	90	0	-	-	-	-	-	-
11	11	22	33	44	55	66	77	88	99	110	0	-	-	-	-	-
12	12	24	36	48	60	72	84	96	108	120	132	0	-	-	-	-
13	13	26	39	52	65	78	91	104	117	130	143	156	0	-	-	-
14	14	28	42	56	70	84	98	112	126	140	154	168	182	0	-	-
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	0	-
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	0
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320
21	21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336
22	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352
23	23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368
24	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384
25	25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400
26	26	52	78	104	130	156	182	208	234	260	286	312	338	364	390	416
27	27	54	81	108	135	162	189	216	243	270	297	324	351	378	405	432
28	28	56	84	112	140	168	196	224	252	280	308	336	364	392	420	448
29	29	58	87	116	145	174	203	232	261	290	319	348	377	406	435	464
30	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450	480
31	31	62	93	124	155	186	217	248	279	310	341	372	403	434	465	496
32	32	64	96	128	160	192	224	256	288	320	352	384	416	448	480	512

TABLE IVc (cont'd)

33	33	66	99	132	165	198	231	264	297	330	363	396	429	462	495	528
34	34	68	102	136	170	204	238	272	306	340	374	408	442	476	510	544
35	35	70	105	140	175	210	245	280	315	350	385	420	455	490	525	560
36	36	72	108	144	180	216	252	288	324	360	396	432	468	504	540	576
37	37	74	111	148	185	222	259	296	333	370	407	444	481	518	555	592
38	38	76	114	152	190	228	266	304	342	380	418	456	494	532	570	608
39	39	78	117	156	195	234	273	312	351	390	429	468	507	546	585	624
40	40	80	120	160	200	240	280	320	360	400	440	480	520	560	600	640
41	41	82	123	164	205	246	287	328	369	410	451	492	533	574	615	656
42	42	84	126	168	210	252	294	336	378	420	462	504	546	588	630	672
43	43	86	129	172	215	258	301	344	387	430	473	516	559	602	645	688
44	44	88	132	176	220	264	308	352	396	440	484	528	572	616	660	704
45	45	90	135	180	225	270	315	360	405	450	495	540	585	630	675	720
46	46	92	138	184	230	276	322	368	414	460	506	552	598	644	690	736
47	47	94	141	188	235	282	329	376	423	470	517	564	611	658	705	752
48	48	96	144	192	240	288	336	384	432	480	528	576	624	672	720	768
49	49	98	147	196	245	294	343	392	441	490	539	588	637	686	735	784
50	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800
51	51	102	153	204	255	306	357	408	459	510	561	612	663	714	765	816
52	52	104	156	208	260	312	364	416	468	520	572	624	676	728	780	832
53	53	106	159	212	265	318	371	424	477	530	583	636	689	742	795	848
54	54	108	162	216	270	324	378	432	486	540	594	648	702	756	810	864
55	55	110	165	220	275	330	385	440	495	550	605	660	715	770	825	880
56	56	112	168	224	280	336	392	448	504	560	616	672	728	784	840	896
57	57	114	171	228	285	342	399	456	513	570	627	684	741	798	855	912
58	58	116	174	232	290	348	406	464	522	580	638	696	754	812	870	928
59	59	118	177	236	295	354	413	472	531	590	649	708	767	826	885	944
60	60	120	180	240	300	360	420	480	540	600	660	720	780	840	900	960
61	61	122	183	244	305	366	427	488	549	610	671	732	793	854	915	976
62	62	124	186	248	310	372	434	496	558	620	682	744	806	868	930	992
63	63	126	189	252	315	378	441	504	567	630	693	756	819	882	945	1008
64	64	128	192	256	320	384	448	512	576	640	704	768	832	896	960	1024

TABLE Va

TOTAL NUMBER S OF SWITCHES FOR UNCONSTRAINED N-TO-M NETWORK (COMPOUND IMPLEMENTATION) ($N \geq M$)

N \ M=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	2	7	0	-	-	-	-	-	-	-	-	-	-	-	-	-
4	3	10	17	0	-	-	-	-	-	-	-	-	-	-	-	-
5	4	13	22	31	0	-	-	-	-	-	-	-	-	-	-	-
6	5	16	25	38	49	0	-	-	-	-	-	-	-	-	-	-
7	6	19	30	41	58	71	0	-	-	-	-	-	-	-	-	-
8	7	22	33	48	61	82	97	0	-	-	-	-	-	-	-	-
9	8	25	38	51	70	85	110	127	0	-	-	-	-	-	-	-
10	9	28	41	58	73	96	113	142	161	0	-	-	-	-	-	-
11	10	31	46	61	80	99	126	145	178	199	0	-	-	-	-	-
12	11	34	49	68	83	106	129	160	181	218	241	0	-	-	-	-
13	12	37	54	71	92	109	136	163	198	221	262	287	0	-	-	-
14	13	40	57	78	95	120	139	170	201	240	265	310	337	0	-	-
15	14	43	62	81	102	123	152	173	208	243	286	313	362	391	0	-
16	15	46	65	88	105	130	155	188	211	250	289	336	365	418	449	0
17	16	49	70	91	114	133	162	191	228	253	296	339	390	421	478	511
18	17	52	73	98	117	144	165	198	231	272	299	346	393	448	481	542
19	18	55	78	101	124	147	178	201	238	275	320	349	400	451	510	545
20	19	58	81	108	127	154	181	216	241	282	323	372	403	458	513	576
21	20	61	86	111	136	157	188	219	256	285	330	375	428	461	520	579
22	21	64	89	118	139	168	191	226	259	300	333	382	431	488	523	586
23	22	67	94	121	146	171	204	229	266	303	348	385	438	491	552	589
24	23	70	97	128	149	178	207	244	269	310	351	400	441	498	555	620
25	24	73	102	131	158	181	214	247	286	313	358	403	456	501	562	623
26	25	76	105	138	161	192	217	254	289	332	361	410	459	516	565	630
27	26	79	110	141	168	195	230	257	296	335	382	413	466	519	580	633
28	27	82	113	148	171	202	233	272	299	342	385	436	469	526	583	648
29	28	85	118	151	180	205	240	275	314	345	392	439	494	529	590	651
30	29	88	121	158	183	216	243	282	317	360	395	446	497	556	593	658
31	30	91	126	161	190	219	256	285	324	363	410	449	504	559	622	661
32	31	94	129	168	193	226	259	300	327	370	413	464	507	566	625	692

TABLE Va (cont'd)

33	32	97	134	171	202	229	266	303	344	373	420	467	522	569	632	695
34	33	100	137	178	205	240	269	310	347	392	423	474	525	584	635	702
35	34	103	142	181	212	243	282	313	354	395	444	477	532	587	650	705
36	35	106	145	188	215	250	285	328	357	402	447	500	535	594	653	720
37	36	109	150	191	224	253	292	331	372	405	454	503	560	597	660	723
38	37	112	153	198	227	264	295	338	375	420	457	510	563	624	663	730
39	38	115	158	201	234	267	308	341	382	423	472	513	570	627	692	733
40	39	118	161	208	237	274	311	356	385	430	475	528	573	634	695	764
41	40	121	166	211	246	277	318	359	402	433	482	531	588	637	702	767
42	41	124	169	218	249	288	321	366	405	452	485	538	591	652	705	774
43	42	127	174	221	256	291	334	369	412	455	506	541	598	655	720	777
44	43	130	177	228	259	298	337	384	415	462	509	564	601	662	723	792
45	44	133	182	231	268	301	344	387	430	465	516	567	626	665	730	795
46	45	136	185	238	271	312	347	394	433	480	519	574	629	692	733	802
47	46	139	190	241	278	315	360	397	440	483	534	577	636	695	762	805
48	47	142	193	248	281	322	363	412	443	490	537	592	639	702	765	836
49	48	145	198	251	290	325	370	415	460	493	544	595	654	705	772	839
50	49	148	201	258	293	336	373	422	463	512	547	602	657	720	775	846
51	50	151	206	261	300	339	386	425	470	515	568	605	664	723	790	849
52	51	154	209	268	303	346	389	440	473	522	571	628	667	730	793	864
53	52	157	214	271	312	349	396	443	488	525	578	631	692	733	800	867
54	53	160	217	278	315	360	399	450	491	540	581	638	695	760	803	874
55	54	163	222	281	322	363	412	453	498	543	596	641	702	763	832	877
56	55	166	225	288	325	370	415	468	501	550	599	656	705	770	835	908
57	56	169	230	291	334	373	422	471	518	553	606	659	720	773	842	911
58	57	172	233	298	337	384	425	478	521	572	609	666	723	788	845	918
59	58	175	238	301	344	387	438	481	528	575	630	669	730	791	860	921
60	59	178	241	308	347	394	441	496	531	582	633	692	733	798	863	936
61	60	181	246	311	356	397	448	499	546	585	640	695	758	801	870	939
62	61	184	249	318	359	408	451	506	549	600	643	702	761	828	873	946
63	62	187	254	321	366	411	464	509	556	603	658	705	768	831	902	949
64	63	190	257	328	369	418	467	524	559	610	661	720	771	838	905	980

TABLE Vb

MAXIMUM NUMBER L OF SWITCHES IN SIGNAL PATH FOR UNCONSTRAINED N-TO-M NETWORK (COMPOUND IMPLEMENTATION) ($N \geq M$)

N \ M=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	2	4	0	-	-	-	-	-	-	-	-	-	-	-	-	-
4	2	5	6	0	-	-	-	-	-	-	-	-	-	-	-	-
5	3	5	8	6	0	-	-	-	-	-	-	-	-	-	-	-
6	3	6	8	8	8	0	-	-	-	-	-	-	-	-	-	-
7	3	6	9	8	10	8	0	-	-	-	-	-	-	-	-	-
8	3	6	9	9	10	10	8	0	-	-	-	-	-	-	-	-
9	4	6	9	9	12	10	10	8	0	-	-	-	-	-	-	-
10	4	7	9	9	12	12	10	10	10	0	-	-	-	-	-	-
11	4	7	10	9	12	12	12	10	12	10	0	-	-	-	-	-
12	4	7	10	10	12	12	12	12	12	12	10	0	-	-	-	-
13	4	7	10	10	13	12	12	12	14	12	12	10	0	-	-	-
14	4	7	10	10	13	13	12	12	14	14	12	12	10	0	-	-
15	4	7	10	10	13	13	13	12	14	14	14	12	12	10	0	-
16	4	7	10	10	13	13	13	13	14	14	14	14	12	12	10	0
17	5	7	10	10	13	13	13	13	16	14	14	14	14	12	12	10
18	5	8	10	10	13	13	13	13	16	16	14	14	14	14	12	12
19	5	8	11	10	13	13	13	13	16	16	16	14	14	14	14	12
20	5	8	11	11	13	13	13	13	16	16	16	16	14	14	14	14
21	5	8	11	11	14	13	13	13	16	16	16	16	16	14	14	14
22	5	8	11	11	14	14	13	13	16	16	16	16	16	16	14	14
23	5	8	11	11	14	14	14	13	16	16	16	16	16	16	16	14
24	5	8	11	11	14	14	14	14	16	16	16	16	16	16	16	16
25	5	8	11	11	14	14	14	14	17	16	16	16	16	16	16	16
26	5	8	11	11	14	14	14	14	17	17	16	16	16	16	16	16
27	5	8	11	11	14	14	14	14	17	17	17	16	16	16	16	16
28	5	8	11	11	14	14	14	14	17	17	17	17	16	16	16	16
29	5	8	11	11	14	14	14	14	17	17	17	17	17	16	16	16
30	5	8	11	11	14	14	14	14	17	17	17	17	17	17	16	16
31	5	8	11	11	14	14	14	14	17	17	17	17	17	17	17	16
32	5	8	11	11	14	14	14	14	17	17	17	17	17	17	17	17

TABLE Vb (cont'd)

33	6	8	11	11	14	14	14	14	17	17	17	17	17	17	17	17
34	6	9	11	11	14	14	14	14	17	17	17	17	17	17	17	17
35	6	9	12	11	14	14	14	14	17	17	17	17	17	17	17	17
36	6	9	12	12	14	14	14	14	17	17	17	17	17	17	17	17
37	6	9	12	12	15	14	14	14	17	17	17	17	17	17	17	17
38	6	9	12	12	15	15	14	14	17	17	17	17	17	17	17	17
39	6	9	12	12	15	15	15	14	17	17	17	17	17	17	17	17
40	6	9	12	12	15	15	15	15	17	17	17	17	17	17	17	17
41	6	9	12	12	15	15	15	15	18	17	17	17	17	17	17	17
42	6	9	12	12	15	15	15	15	18	18	17	17	17	17	17	17
43	6	9	12	12	15	15	15	15	18	18	18	17	17	17	17	17
44	6	9	12	12	15	15	15	15	18	18	18	18	17	17	17	17
45	6	9	12	12	15	15	15	15	18	18	18	18	18	17	17	17
46	6	9	12	12	15	15	15	15	18	18	18	18	18	18	17	17
47	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	17
48	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
49	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
50	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
51	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
52	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
53	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
54	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
55	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
56	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
57	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
58	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
59	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
60	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
61	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
62	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
63	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18
64	6	9	12	12	15	15	15	15	18	18	18	18	18	18	18	18

TABLE Vc

MAXIMUM NUMBER P OF PHYSICAL SIGNAL PATHS FOR UNCONSTRAINED N-TO-M NETWORK (COMPONENT IMPLEMENTATION) ($N \geq M$)

N \ M=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	2	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	3	4	0	-	-	-	-	-	-	-	-	-	-	-	-	-
4	4	6	9	0	-	-	-	-	-	-	-	-	-	-	-	-
5	5	8	9	16	0	-	-	-	-	-	-	-	-	-	-	-
6	6	10	10	16	25	0	-	-	-	-	-	-	-	-	-	-
7	7	12	12	16	25	36	0	-	-	-	-	-	-	-	-	-
8	8	14	14	16	25	36	49	0	-	-	-	-	-	-	-	-
9	9	16	16	16	25	36	49	64	0	-	-	-	-	-	-	-
10	10	18	18	18	25	36	49	64	81	0	-	-	-	-	-	-
11	11	20	20	20	25	36	49	64	81	100	0	-	-	-	-	-
12	12	22	22	22	25	36	49	64	81	100	121	0	-	-	-	-
13	13	24	24	24	25	36	49	64	81	100	121	144	0	-	-	-
14	14	26	26	26	26	36	49	64	81	100	121	144	169	0	-	-
15	15	28	28	28	28	36	49	64	81	100	121	144	169	196	0	-
16	16	30	30	30	30	36	49	64	81	100	121	144	169	196	225	0
17	17	32	32	32	32	36	49	64	81	100	121	144	169	196	225	256
18	18	34	34	34	34	36	49	64	81	100	121	144	169	196	225	256
19	19	36	36	36	36	36	49	64	81	100	121	144	169	196	225	256
20	20	38	38	38	38	38	49	64	81	100	121	144	169	196	225	256
21	21	40	40	40	40	40	49	64	81	100	121	144	169	196	225	256
22	22	42	42	42	42	42	49	64	81	100	121	144	169	196	225	256
23	23	44	44	44	44	44	49	64	81	100	121	144	169	196	225	256
24	24	46	46	46	46	46	49	64	81	100	121	144	169	196	225	256
25	25	48	48	48	48	48	49	64	81	100	121	144	169	196	225	256
26	26	50	50	50	50	50	50	64	81	100	121	144	169	196	225	256
27	27	52	52	52	52	52	52	64	81	100	121	144	169	196	225	256
28	28	54	54	54	54	54	54	64	81	100	121	144	169	196	225	256
29	29	56	56	56	56	56	56	64	81	100	121	144	169	196	225	256
30	30	58	58	58	58	58	58	64	81	100	121	144	169	196	225	256
31	31	60	60	60	60	60	60	64	81	100	121	144	169	196	225	256
32	32	62	62	62	62	62	62	64	81	100	121	144	169	196	225	256

TABLE Vc (cont'd)

33	33	64	64	64	64	64	64	64	81	100	121	144	169	196	225	256
34	34	66	66	66	66	66	66	66	81	100	121	144	169	196	225	256
35	35	68	68	68	68	68	68	68	81	100	121	144	169	196	225	256
36	36	70	70	70	70	70	70	70	81	100	121	144	169	196	225	256
37	37	72	72	72	72	72	72	72	81	100	121	144	169	196	225	256
38	38	74	74	74	74	74	74	74	81	100	121	144	169	196	225	256
39	39	76	76	76	76	76	76	76	81	100	121	144	169	196	225	256
40	40	78	78	78	78	78	78	78	81	100	121	144	169	196	225	256
41	41	80	80	80	80	80	80	80	81	100	121	144	169	196	225	256
42	42	82	82	82	82	82	82	82	82	100	121	144	169	196	225	256
43	43	84	84	84	84	84	84	84	84	100	121	144	169	196	225	256
44	44	86	86	86	86	86	86	86	86	100	121	144	169	196	225	256
45	45	88	88	88	88	88	88	88	88	100	121	144	169	196	225	256
46	46	90	90	90	90	90	90	90	90	100	121	144	169	196	225	256
47	47	92	92	92	92	92	92	92	92	100	121	144	169	196	225	256
48	48	94	94	94	94	94	94	94	94	100	121	144	169	196	225	256
49	49	96	96	96	96	96	96	96	96	100	121	144	169	196	225	256
50	50	98	98	98	98	98	98	98	98	100	121	144	169	196	225	256
51	51	100	100	100	100	100	100	100	100	100	121	144	169	196	225	256
52	52	102	102	102	102	102	102	102	102	102	121	144	169	196	225	256
53	53	104	104	104	104	104	104	104	104	104	121	144	169	196	225	256
54	54	106	106	106	106	106	106	106	106	106	121	144	169	196	225	256
55	55	108	108	108	108	108	108	108	108	108	121	144	169	196	225	256
56	56	110	110	110	110	110	110	110	110	110	121	144	169	196	225	256
57	57	112	112	112	112	112	112	112	112	112	121	144	169	196	225	256
58	58	114	114	114	114	114	114	114	114	114	121	144	169	196	225	256
59	59	116	116	116	116	116	116	116	116	116	121	144	169	196	225	256
60	60	118	118	118	118	118	118	118	118	118	121	144	169	196	225	256
61	61	120	120	120	120	120	120	120	120	120	121	144	169	196	225	256
62	62	122	122	122	122	122	122	122	122	122	122	144	169	196	225	256
63	63	124	124	124	124	124	124	124	124	124	124	144	169	196	225	256
64	64	126	126	126	126	126	126	126	126	126	126	144	169	196	225	256

TABLE VIa

TOTAL NUMBER S OF SWITCHES FOR ORDER-CONSTRAINED N-TO-M NETWORK ($N \geq M$)

N \ M=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	2	3	0	-	-	-	-	-	-	-	-	-	-	-	-	-
4	3	6	5	0	-	-	-	-	-	-	-	-	-	-	-	-
5	4	9	10	7	0	-	-	-	-	-	-	-	-	-	-	-
6	5	12	13	14	9	0	-	-	-	-	-	-	-	-	-	-
7	6	15	18	17	18	11	0	-	-	-	-	-	-	-	-	-
8	7	18	21	24	21	22	13	0	-	-	-	-	-	-	-	-
9	8	21	26	27	30	25	26	15	0	-	-	-	-	-	-	-
10	9	24	29	34	33	36	29	30	17	0	-	-	-	-	-	-
11	10	27	34	37	40	39	42	33	34	19	0	-	-	-	-	-
12	11	30	37	44	43	46	45	48	37	38	21	0	-	-	-	-
13	12	33	42	47	52	49	52	51	.54	41	42	23	0	-	-	-
14	13	36	45	54	55	60	55	58	57	60	45	46	25	0	-	-
15	14	39	50	57	62	63	68	61	64	63	66	49	50	27	0	-
16	15	42	53	64	65	70	71	76	67	70	69	72	53	54	29	0
17	16	45	58	67	74	73	78	79	84	73	76	75	78	57	58	31
18	17	48	61	74	77	84	81	86	87	92	79	82	81	84	61	62
19	18	51	66	77	84	87	94	89	94	95	100	85	88	87	90	65
20	19	54	69	84	87	94	97	104	97	102	103	108	91	94	93	96
21	20	57	74	87	96	97	104	107	112	105	110	111	116	97	100	99
22	21	60	77	94	99	108	107	114	115	120	113	118	119	124	103	106
23	22	63	82	97	106	111	120	117	122	123	128	121	126	127	132	109
24	23	66	85	104	109	118	123	132	125	130	131	136	129	134	135	140
25	24	69	90	107	118	121	130	135	142	133	138	139	144	137	142	143
26	25	72	93	114	121	132	133	142	145	152	141	146	147	152	145	150
27	26	75	98	117	128	135	146	145	152	155	162	149	154	155	160	153
28	27	78	101	124	131	142	149	160	155	162	165	172	157	162	163	168
29	28	81	106	127	140	145	156	163	170	165	172	175	182	165	170	171
30	29	84	109	134	143	156	159	170	173	180	175	182	185	192	173	178
31	30	87	114	137	150	159	172	173	180	183	190	185	192	195	202	181
32	31	90	117	144	153	166	175	188	183	190	193	200	195	202	205	212

TABLE VIa (cont'd)

33	32	93	122	147	162	169	182	191	200	193	200	203	210	205	212	215
34	33	96	125	154	165	180	185	198	203	212	203	210	213	220	215	222
35	34	99	130	157	172	183	198	201	210	215	224	213	220	223	230	225
36	35	102	133	164	175	190	201	216	213	222	227	236	223	230	233	240
37	36	105	138	167	184	193	208	219	228	225	234	239	248	233	240	243
38	37	108	141	174	187	204	211	226	231	240	237	246	251	260	243	250
39	38	111	146	177	194	207	224	229	238	243	252	249	258	263	272	253
40	39	114	149	184	197	214	227	244	241	250	255	264	261	270	275	284
41	40	117	154	187	206	217	234	247	258	253	262	267	276	273	282	287
42	41	120	157	194	209	228	237	254	261	272	265	274	279	288	285	294
43	42	123	162	197	216	231	250	257	268	275	286	277	286	291	300	297
44	43	126	165	204	219	238	253	272	271	282	289	300	289	298	303	312
45	44	129	170	207	228	241	260	275	286	285	296	303	314	301	310	315
46	45	132	173	214	231	252	263	282	289	300	299	310	317	328	313	322
47	46	135	178	217	238	255	276	285	296	303	314	313	324	331	342	325
48	47	138	181	224	241	262	279	300	299	310	317	328	327	338	345	356
49	48	141	186	227	250	265	286	303	316	313	324	331	342	341	352	359
50	49	144	189	234	253	276	289	310	319	332	327	338	345	356	355	366
51	50	147	194	237	260	279	302	313	326	335	348	341	352	359	370	369
52	51	150	197	244	263	286	305	328	329	342	351	364	355	366	373	384
53	52	153	202	247	272	289	312	331	344	345	358	367	380	369	380	387
54	53	156	205	254	275	300	315	338	347	360	361	374	383	396	383	394
55	54	159	210	257	282	303	328	341	354	363	376	377	390	399	412	397
56	55	162	213	264	285	310	331	356	357	370	379	392	393	406	415	428
57	56	165	218	267	294	313	338	359	374	373	386	395	408	409	422	431
58	57	168	221	274	297	324	341	366	377	392	389	402	411	424	425	438
59	58	171	226	277	304	327	354	369	384	395	410	405	418	427	440	441
60	59	174	229	284	307	334	357	384	387	402	413	428	421	434	443	456
61	60	177	234	287	316	337	364	387	402	405	420	431	446	437	450	459
62	61	180	237	294	319	348	367	394	405	420	423	438	449	464	453	466
63	62	183	242	297	326	351	380	397	412	423	438	441	456	467	482	469
64	63	186	245	304	329	358	383	412	415	430	441	456	459	474	485	500

TABLE VIb

MAXIMUM NUMBER L OF SWITCHES IN SIGNAL PATH FOR ORDER-CONSTRAINED N-TO-M NETWORK ($N \geq M$)

N \ M=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	2	2	0	-	-	-	-	-	-	-	-	-	-	-	-	-
4	2	3	2	0	-	-	-	-	-	-	-	-	-	-	-	-
5	3	3	4	2	0	-	-	-	-	-	-	-	-	-	-	-
6	3	4	4	4	2	0	-	-	-	-	-	-	-	-	-	-
7	3	4	5	4	4	2	0	-	-	-	-	-	-	-	-	-
8	3	4	5	5	4	4	2	0	-	-	-	-	-	-	-	-
9	4	4	5	5	6	4	4	2	0	-	-	-	-	-	-	-
10	4	5	5	5	6	6	4	4	2	0	-	-	-	-	-	-
11	4	5	6	5	6	6	6	4	4	2	0	-	-	-	-	-
12	4	5	6	6	6	6	6	6	4	4	2	0	-	-	-	-
13	4	5	6	6	7	6	6	6	6	4	4	2	0	-	-	-
14	4	5	6	6	7	7	6	6	6	6	4	4	2	0	-	-
15	4	5	6	6	7	7	7	6	6	6	6	4	4	2	0	-
16	4	5	6	6	7	7	7	7	6	6	6	6	4	4	2	0
17	5	5	6	6	7	7	7	7	8	6	6	6	6	4	4	2
18	5	6	6	6	7	7	7	7	8	8	6	6	6	6	4	4
19	5	6	7	6	7	7	7	7	8	8	8	6	6	6	6	4
20	5	6	7	7	7	7	7	7	8	8	8	8	6	6	6	6
21	5	6	7	7	8	7	7	7	8	8	8	8	8	6	6	6
22	5	6	7	7	8	8	7	7	8	8	8	8	8	8	6	6
23	5	6	7	7	8	8	8	7	8	8	8	8	8	8	8	6
24	5	6	7	7	8	8	8	8	8	8	8	8	8	8	8	8
25	5	6	7	7	8	8	8	8	9	8	8	8	8	8	8	8
26	5	6	7	7	8	8	8	8	9	9	8	8	8	8	8	8
27	5	6	7	7	8	8	8	8	9	9	9	8	8	8	8	8
28	5	6	7	7	8	8	8	8	9	9	9	9	8	8	8	8
29	5	6	7	7	8	8	8	8	9	9	9	9	9	8	8	8
30	5	6	7	7	8	8	8	8	9	9	9	9	9	9	8	8
31	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	8
32	5	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9

TABLE VIb (cont'd)

33	6	6	7	7	8	8	8	8	9	9	9	9	9	9	9	9
34	6	7	7	7	8	8	8	8	9	9	9	9	9	9	9	9
35	6	7	8	7	8	8	8	8	9	9	9	9	9	9	9	9
36	6	7	8	8	8	8	8	8	9	9	9	9	9	9	9	9
37	6	7	8	8	9	8	8	8	9	9	9	9	9	9	9	9
38	6	7	8	8	9	9	8	8	9	9	9	9	9	9	9	9
39	6	7	8	8	9	9	9	8	9	9	9	9	9	9	9	9
40	6	7	8	8	9	9	9	9	9	9	9	9	9	9	9	9
41	6	7	8	8	9	9	9	9	10	9	9	9	9	9	9	9
42	6	7	8	8	9	9	9	9	10	10	9	9	9	9	9	9
43	6	7	8	8	9	9	9	9	10	10	10	9	9	9	9	9
44	6	7	8	8	9	9	9	9	10	10	10	10	9	9	9	9
45	6	7	8	8	9	9	9	9	10	10	10	10	10	9	9	9
46	6	7	8	8	9	9	9	9	10	10	10	10	10	10	9	9
47	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	9
48	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
49	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
50	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
51	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
52	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
53	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
54	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
55	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
56	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
57	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
58	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
59	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
60	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
61	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
62	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
63	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10
64	6	7	8	8	9	9	9	9	10	10	10	10	10	10	10	10

TABLE VIc

MAXIMUM NUMBER P OF PHYSICAL SIGNAL PATHS FOR ORDER-CONSTRAINED N-TO-M NETWORK (N $\leq$ M)

N \ M=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	2	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	3	4	0	-	-	-	-	-	-	-	-	-	-	-	-	-
4	4	6	6	0	-	-	-	-	-	-	-	-	-	-	-	-
5	5	8	8	8	0	-	-	-	-	-	-	-	-	-	-	-
6	6	10	10	10	10	0	-	-	-	-	-	-	-	-	-	-
7	7	12	12	12	12	12	0	-	-	-	-	-	-	-	-	-
8	8	14	14	14	14	14	14	0	-	-	-	-	-	-	-	-
9	9	16	16	16	16	16	16	16	0	-	-	-	-	-	-	-
10	10	18	18	18	18	18	18	18	18	0	-	-	-	-	-	-
11	11	20	20	20	20	20	20	20	20	20	0	-	-	-	-	-
12	12	22	22	22	22	22	22	22	22	22	22	0	-	-	-	-
13	13	24	24	24	24	24	24	24	24	24	24	24	0	-	-	-
14	14	26	26	26	26	26	26	26	26	26	26	26	26	0	-	-
15	15	28	28	28	28	28	28	28	28	28	28	28	28	28	0	-
16	16	30	30	30	30	30	30	30	30	30	30	30	30	30	30	0
17	17	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32
18	18	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34
19	19	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36
20	20	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38
21	21	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
22	22	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42
23	23	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44
24	24	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
25	25	48	48	48	48	48	48	48	48	48	48	48	48	48	48	48
26	26	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
27	27	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52
28	28	54	54	54	54	54	54	54	54	54	54	54	54	54	54	54
29	29	56	56	56	56	56	56	56	56	56	56	56	56	56	56	56
30	30	58	58	58	58	58	58	58	58	58	58	58	58	58	58	58
31	31	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60
32	32	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62

TABLE VIc (cont'd)

33	33	64	64	64	64	64	64	64	64	64	64	64	64	64	64	64
34	34	66	66	66	66	66	66	66	66	66	66	66	66	66	66	66
35	35	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68
36	36	70	70	70	70	70	70	70	70	70	70	70	70	70	70	70
37	37	72	72	72	72	72	72	72	72	72	72	72	72	72	72	72
38	38	74	74	74	74	74	74	74	74	74	74	74	74	74	74	74
39	39	76	76	76	76	76	76	76	76	76	76	76	76	76	76	76
40	40	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78
41	41	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80
42	42	82	82	82	82	82	82	82	82	82	82	82	82	82	82	82
43	43	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84
44	44	86	86	86	86	86	86	86	86	86	86	86	86	86	86	86
45	45	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
46	46	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
47	47	92	92	92	92	92	92	92	92	92	92	92	92	92	92	92
48	48	94	94	94	94	94	94	94	94	94	94	94	94	94	94	94
49	49	96	96	96	96	96	96	96	96	96	96	96	96	96	96	96
50	50	98	98	98	98	98	98	98	98	98	98	98	98	98	98	98
51	51	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
52	52	102	102	102	102	102	102	102	102	102	102	102	102	102	102	102
53	53	104	104	104	104	104	104	104	104	104	104	104	104	104	104	104
54	54	106	106	106	106	106	106	106	106	106	106	106	106	106	106	106
55	55	108	108	108	108	108	108	108	108	108	108	108	108	108	108	108
56	56	110	110	110	110	110	110	110	110	110	110	110	110	110	110	110
57	57	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112
58	58	114	114	114	114	114	114	114	114	114	114	114	114	114	114	114
59	59	116	116	116	116	116	116	116	116	116	116	116	116	116	116	116
60	60	118	118	118	118	118	118	118	118	118	118	118	118	118	118	118
61	61	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120
62	62	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122
63	63	124	124	124	124	124	124	124	124	124	124	124	124	124	124	124
64	64	126	126	126	126	126	126	126	126	126	126	126	126	126	126	126

VII. Acknowledgment

I would like to thank Bill Cummings for his comments on waveguide switch networks and Ben Eaves for his discussions on queuing.

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multiple-beam antennas
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

There exist three classes of RF beam-switching networks for creating, with a multiple-beam antenna (MBA), a set of electronically steerable antenna beams. For a 61-beam receive MBA with eight steered beams, for example, these classes can connect the eight output ports to the selected eight of the 61-beam feeds, with a) no constraint on selection or order of the eight beam feeds, b) contraints on the order alone, or c) contraints on both selection and order. The total number of switches required for this is 907, 387, or 175, respectively, indicating substantial differences in complexity between classes, and yet they can be designed to have very similar traffic performance.

The report examines the general design, performance, and complexity of such networks. Included are two further measures of complexity as well as the switching algorithm and the effect of non-uniform traffic.

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